Calculus 3208 **Name:** ____________________

Review Sheet: Applications of Derivatives

1. Given the following function $f(x) = 4 - 3x^2 + x^3$, answer the following questions:

   (a) Determine the x and y-intercepts of the graph of $f(x)$, if any.

   (b) Determine the intervals on which $f(x)$ is increasing or decreasing and classify any relative (local) extrema.

   (c) Determine the intervals on which $f(x)$ is concave upward or concave downward and identify any inflection points.

   (d) Sketch the graph of $f(x)$. Label any inflection points and extrema.

Xint $(y=0)$ $0 = x^3 - 3x^2 + 4$

$$\begin{array}{r|rrrr} -1 & 1 & -3 & 0 & 4 \\ & & -1 & 4 & -4 \\ \hline & 1 & -4 & 4 & 0R \end{array}$$

$x^2 - 4x + 4 = 0$

$(x-2)(x-2) = 0$

$x = 2$ double

$f'(x) = 3x^2 - 6x$

$0 = 3x(x-2)$

$x = 0$ $x = 2$

$+\nearrow$, $-\searrow$, $+\nearrow$

0 2

local max $(0,4)$

local min $(2,0)$

$f''(x) = 6x - 6$

$6x = 6$

$x = 1$

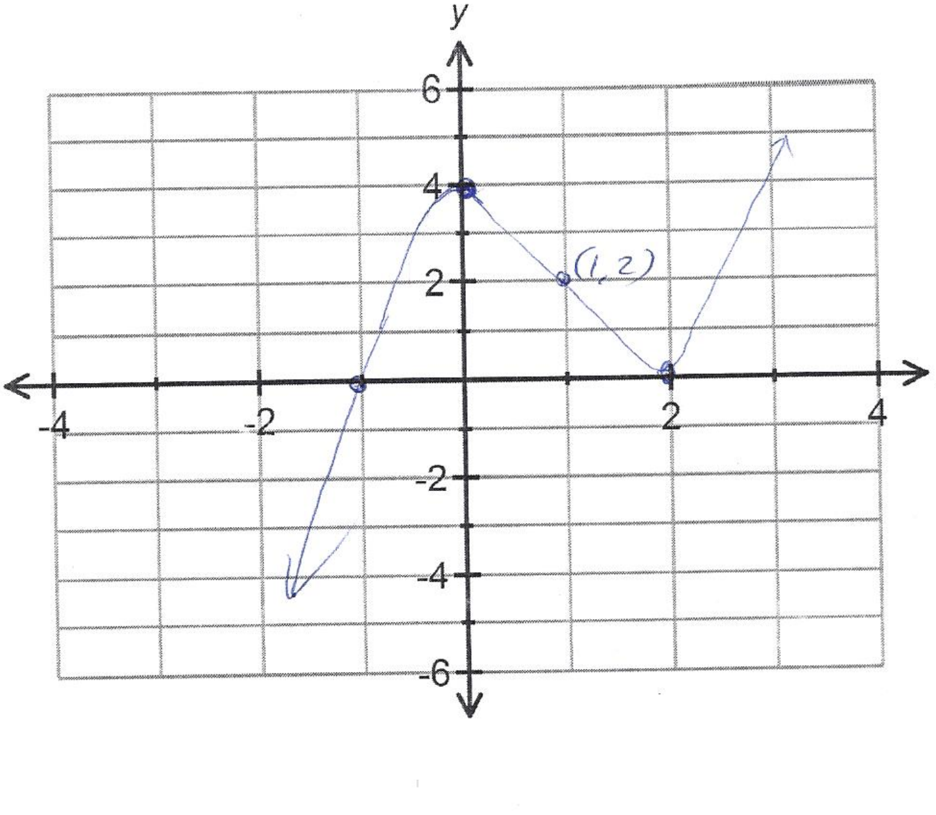
$-$ $+$

$\frown$ 1 $\smile$

Point Infl. $(1,2)$

yint $(x=0)$ $y = 4$

No Asymptotes



2. Given the following function with the indicated derivatives, answer the following questions:

$$f(x)=\frac{x^2+1}{x^2-1};\quad f'(x)=\frac{-4x}{(x^2-1)^2};\quad f''(x)=\frac{4(3x^2+1)}{(x^2-1)^3}$$

(a) Determine the vertical and horizontal asymptotes of $f(x)$, if any.

(b) Determine the x and y-intercepts of the graph of $f(x)$, if any.

(c) Determine the intervals on which $f(x)$ is increasing or decreasing and classify any relative (local) extrema.

(d) Determine the intervals on which $f(x)$ is concave upward or concave downward and identify any inflection points.

(e) Sketch the graph of $f(x)$. Label any inflection points and extrema.

VA $\boxed{x = \pm 1}$

HA $\lim_{x \to \pm\infty} f(x) = 1$ $\boxed{y=1}$

xint $x^2+1=0$ $\boxed{\text{no xint}}$

yint $\boxed{y=-1}$

$f''(x)=0$

$4(3x^2+1)=0$

$3x^2+1=0$

$3x^2=-1$ no solution

$+ \quad - \quad +$

-1 VA $\quad 1$ VA

$f'(x)=0$

$-4x=0$

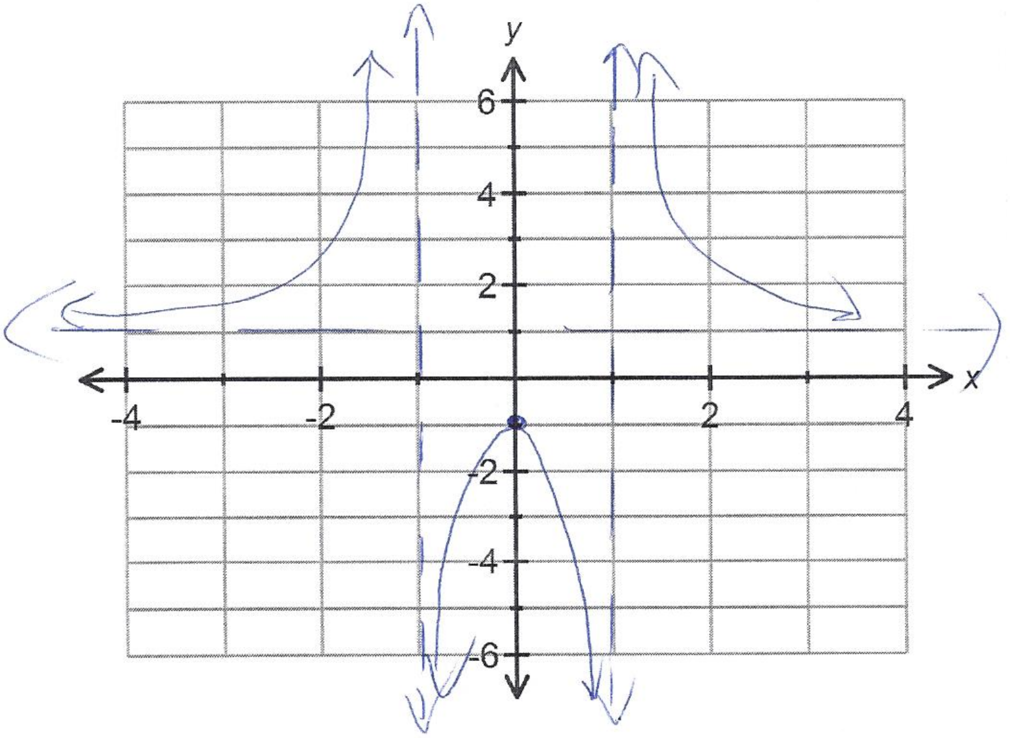
$x=0$

$+\uparrow \quad +\uparrow \quad -\searrow \quad -\searrow$

$-1 \quad 0 \quad 1$

VA $\quad$ VA

$(0,-1)$ local max



f(x) does not cross HA

3. A snowball is melting such that the radius is decreasing by 0.15cm/min. How fast is the volume changing when the radius is 6 cm? $\left[V = \frac{4}{3}\pi r^3\right]$

sphere

$\frac{dr}{dt} = -0.15$ cm/min

$\frac{dV}{dt} = ?$ $r = 6$cm

$V = \frac{4}{3}\pi r^3$

$\frac{dV}{dt} = \frac{4}{\cancel{3}}\pi\left[\cancel{3}r^2 \frac{dr}{dt}\right]$

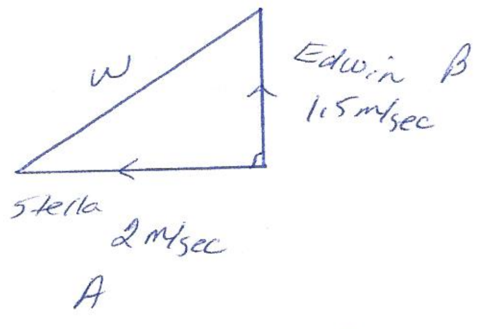
$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

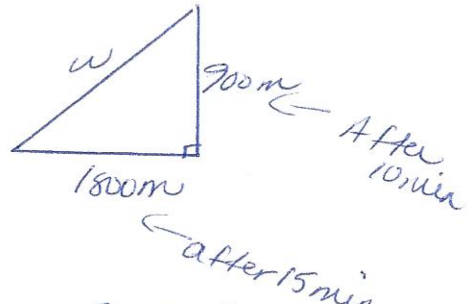
$\frac{dV}{dt} = 4\pi (6)^2 (-0.15\text{ cm/min})$

$\frac{dV}{dt} = -67.9 \frac{\text{cm}^3}{\text{min}}$

4. Edwin and Stella part ways at a crossroads. Stella moodily runs to the west, moving at a constant rate of 2 metres per second. Edwin glumly watches her vanish into the distance for 5 minutes before he turns to the north and heads in that direction at a constant rate of 1.5 metres per second. Determine how quickly the distance between them is increasing **15 minutes** after Stella's departure.

15min =
15 × 60
= 900sec





$w^2 = 1800^2 + 900^2$

$w^2 = 4050000$

$w = 2012.46$m

$w^2 = A^2 + B^2$

$\cancel{2}w \frac{dw}{dt} = \cancel{2}A \frac{dA}{dt} + \cancel{2}B \frac{dB}{dt}$

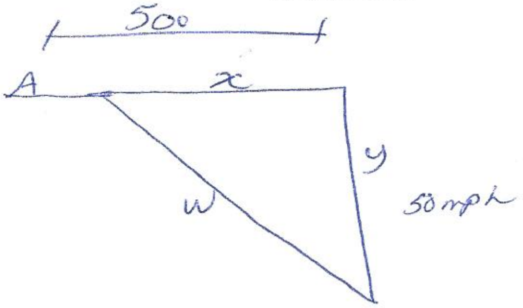
$2012.46 \frac{dw}{dt} = 1800(2\text{m/sec}) + 900(1.5)$

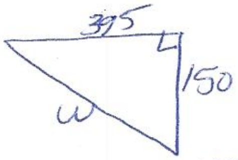
$2012.46 \frac{dw}{dt} = 4950$

$\frac{dw}{dt} = 2.5$ m/sec

5. Two cars start out 500 miles apart. Car A is to the west of Car B and starts driving to the east (*i.e.* towards Car B) at 35 mph and at the same time Car B starts driving south at 50 mph. After 3 hours of driving at what rate is the distance between the two cars changing? Is it increasing or decreasing?

x → distance separating Car A and car B





$w = 422.5$

$$x^2 + y^2 = w^2$$

$$\cancel{2}x\frac{dx}{dt} + \cancel{2}y\frac{dy}{dt} = \cancel{2}w\frac{dw}{dt}$$

$$395(-35) + (150)50 = 422.5\frac{dw}{dt}$$

$$-6325 = 422.5\frac{dw}{dt}$$

$$\frac{dw}{dt} = -14.97 \text{ mph}$$

6. Sand falling from a chute forms a conical pile whose height is always equal to 4/3 of the radius of the base. How fast is the volume changing when the radius of the base is 36 inches and is increasing at a rate of 3 in/min? $\left[V = \frac{1}{3}\pi r^2 h\right]$ $\frac{dv}{dt} = ?$ $h = \frac{4}{3}r$ $\frac{dr}{dt} = 3\frac{in}{min}$

$$r = \frac{1}{3}\pi r^2 h$$

$$v = \frac{1}{3}\pi r^2\left(\frac{4}{3}r\right)$$

$$v = \frac{1}{3}\pi r^3\left(\frac{4}{3}\right)$$

$$v = \frac{4}{9}\pi r^3$$

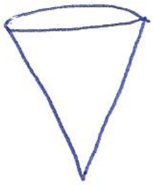
$$\frac{dv}{dt} = \frac{4}{9}\pi\left[3r^2\frac{dr}{dt}\right]$$

$$\frac{dv}{dt} = \frac{4}{3}\pi r^2\frac{dr}{dt}$$

$$\frac{dv}{dt} = \frac{4}{3}\pi(36)(3) = 16286.02 \text{ in}^3/\text{min}$$

7. An ice sculpture in the shape of an inverted cone is melting such that its height shrinks at a rate of $\frac{1}{3}$ metres per hour but its radius remains constant. How fast is the cone's voulume decreasing when its height is 1 metre and its volume is 3π cubic metres?

$V = \frac{1}{3}\pi r^2 h$



$$\frac{dV}{dt} = \frac{1}{3}\pi\left[r^2\frac{dh}{dt} + h\left(2r \cdot \underset{\to 0}{\frac{dr}{dt}}\right)\right]$$

$$\frac{dV}{dt} = \frac{1}{3}\pi\left[r^2\right]\frac{dh}{dt}$$

$\frac{dh}{dt} = -\frac{1}{3}$ m/hr

radius does not change $\frac{dr}{dt} = 0$

$$\frac{dV}{dt} = \frac{1}{3}\pi(3)^2\left(-\frac{1}{3}\right)$$

$$\frac{dV}{dt} = \frac{1}{3}\pi(9)\left(-\frac{1}{3}\right)$$

$$\frac{dV}{dt} = -\pi \text{ m}^3/\text{hr}$$

$\left.\begin{array}{l} h = 1 \\ V = 3\pi \end{array}\right\}$ $3\pi = \frac{1}{3}\pi r^2(1)$

$9\cancel{\pi} = \cancel{\pi} r^2$

$3 = r$

8. A box with a square base and open top is to be made with $1200\,cm^2$ of cardboard. Determine the dimensions which will give the largest possible volume of the box.

Area

MAX

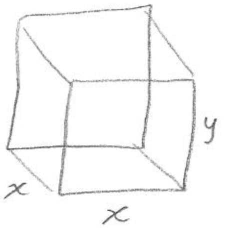
$\begin{cases} V = l \cdot w \cdot h \\ V = x \cdot x \cdot y = x^2 y \end{cases}$

$SA = x^2 + 4xy$

$1200 = x^2 + 4xy$

$1200 - x^2 = 4xy$

$\frac{1200 - x^2}{4x} = y$



$V = x^2 y$

$V = x^2\left(\frac{1200 - x^2}{4x}\right)$

$V = \frac{(1200 - x^2)x}{4} = \frac{1200x - x^3}{4}$

$V = 300x - \frac{1}{4}x^3$

$V' = 300 - \frac{3}{4}x^2$ $\qquad V'' = -\frac{6}{4}x$

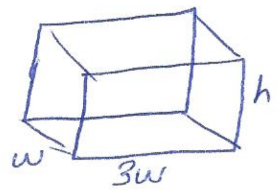
$0 = 300 - \frac{3}{4}x^2$ $\qquad V''(20) < 0$

$\frac{3}{4}x^2 = 300$ $\qquad \Rightarrow$ maximum

$x^2 = 400$

$\begin{cases} x = 20 \text{ cm} \\ y = \frac{1200 - 400}{80} = 10 \text{ cm} \end{cases}$

9. We want to construct a box whose base length is 3 times the base width. The material used to build the top and bottom cost \$10/ft^2 and the material used to build the sides cost \$6/ft^2. If the box must have a volume of 50 ft^3, determine the dimensions that will minimize the cost to build the box.



$V = l \cdot w \cdot h$

$50 = 3w(w)(h)$

$50 = 3w^2 h$

$\frac{50}{3w^2} = h$

$SA = 3w^2 + 3w^2 + 6wh + 2wh$

$SA = 6w^2 + 8wh$

$C = 10(6w^2) + 6(8wh)$

$C = 60w^2 + 48wh$

$C = 60w^2 + 48w\left(\frac{50}{3w^2}\right)$

$C = 60w^2 + \frac{800}{w}$

$C = 60w^2 + 800w^{-1}$

$C' = 120w - 800w^{-2}$

$0 = 120w - \frac{800}{w^2}$

$0 = \frac{120w^3 - 800}{w^2} \Rightarrow$

$120w^3 = 800$

$w^3 = 20/3$

$w = \sqrt[3]{20/3}$

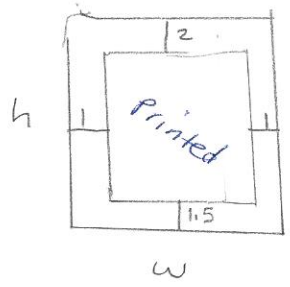
$C'' = 120 + 1600w^{-3}$

$C'' = 120 + \frac{1600}{w^3}$

$C''\left(\sqrt[3]{\frac{20}{3}}\right) > 0$

Minimum

10. A printer needs to make a poster that will have a total area of 200 in^2 and will have 1 inch margins on the sides, a 2 inch margin on the top and a 1.5 inch margin on the bottom. What dimensions will give the largest printed area (the area of the poster with the margins taken out)?



$200 = wh$

$\frac{200}{w} = h$

$Printed = (w-2)(h-3.5)$

$A = (w-2)\left(\frac{200}{w} - 3.5\right)$

$A = 200 - 3.5w - \frac{400}{w} + 7$

$A' = -3.5 + \frac{400}{w^2}$

$0 = \frac{-3.5w^2 + 400}{w^2}$

$0 = -3.5w^2 + 400$

$3.5w^2 = 400$

$w^2 = \frac{400}{3.5}$

$w = \sqrt{\frac{400}{3.5}}$

$A'' = -\frac{800}{w^3}$

$A''\left(\sqrt{\frac{400}{3.5}}\right) = \ominus < 0$

MAXIMUM