Section A: 18 Short Multiple Choice (1 mark each = 18 marks)

Section B: 5 Long Answer Multiple Choice (4 marks each = 20 marks)→ Must show workings.

Section C: 4 Long Answer Questions (30 marks)→ Must show workings.

Quadratic Formula: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Unit 1: Pre-Calculus

1. Function Operations:

Sum	$f(x)+g(x)$	Domain and Range
Difference	$f(x)-g(x)$	
Product	$f(x)\times g(x)$	
Quotient	$\frac{f(x)}{g(x)}$	

Quotient: Non-permissible values → produce vertical asymptotes or points of discontinuity

2. Composite Functions $f(g(x))$ **and finding its domain**

Unit 2: Limits and Continuity

1. Limit Definition and Notation

$\lim_{x \to a} f(x) = L$ → We can make $f(x)$ as close to L by making x close to a on either side but not equal to a.

2. Left and Right Hand Limits using Piece-wise Function

$\lim_{x \to a} f(x) = L$ if and only if $\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x) = L$

3. Evaluate Limits of a *Polynomial Function*

$\lim_{x \to a} f(x) = f(a)$ → simply substitute in the value of x being approached.

4. Evaluate Limits of a *Rational Function*
(using substitution)

A. $\frac{0}{0}$ → Indeterminate Form

Simplify the function by factoring, using conjugates, obtain the lowest common denominator or expand the function.

B. $\frac{c}{0}$ → Infinite Limit

$x = a$ Is a vertical asymptote

$\lim_{x \to a} f(x) = \pm\infty$

5. Limits using absolute Value

→ evaluate using left and right hand limits since the absolute function is a piecewise function.

6. Formula for factoring a sum/Difference of cubes:

$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$ $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

7. Limits at Infinity

$\lim_{x \to \pm\infty} f(x) = b$ → y = b is a horizontal asymptote

→ divide the numerator and denominator by the highest power of x appearing in the denominator

$\lim_{x \to \pm\infty} \frac{1}{x^n} = 0$

8. Oblique Asymptotes

→ when the degree of numerator is 1 more than the degree of the denominator

→ the equation of the oblique asymptote is the quotient resulting from synthetic division.

9. Definition of Continuity

→ three step process to prove continuity:

$(i)\quad f(a) \text{ is defined}$

$(ii)\quad \lim_{x \to a} f(x) \text{ exists}$

$(iii)\quad f(a) = \lim_{x \to a} f(x)$

10. Types of Discontinuity

A. Removable Discontinuity
B. Non-Removable Discontinuity (jump and infinite)

→ To prove discontinuities, test points where the function is changing for a piecewise function or the non-permissible values of the rational function.

- ➔ Removable discontinuity: limit exists but $f(a) \neq \lim_{x \to a} f(x)$
- ➔ Non-Removable discontinuity: limit does not exist $\lim_{x \to a^+} f(x) \neq \lim_{x \to a^-} f(x)$

Unit 3: Rational Functions

1. <u>Graphing a rational function</u>

Identify the following:

a. x and y-intercepts

b. non-permissible values

c. vertical asymptotes

d. points of discontinuity

e. horizontal asymptotes

f. oblique asymptote

g. check if the function crosses the horizontal asymptote

h. sign diagram

i. graph (use a few test points)

Note:

- When a graph has a hole in it (or a point of discontinuity), the numerator and denominator of the original function contains a common factor

- A factor of only the denominator results in a vertical asymptote

- A factor of only the numerator results in an x-intercept

- Test points on the sign diagram tells us where the function is positive or negative for different intervals of x.

- Horizontal asymptotes describe the end behavior of the function

- Vertical asymptotes describe the behavior of the function near a point.

Unit 4: Derivatives

1. Derivative

→ instantaneous rate of change

→ Slope of a tangent line

2. Definition of the Derivative

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

3. Equation of a tangent line **at** $x = a$

→ $y - f(a) = f'(a)(x - a)$

4. Equation of a normal line **at** $x = a$

→ $y - f(a) = -\frac{1}{f'(a)}(x - a)$

Tangent line and normal line are perpendicular to each other. Their slopes are negative reciprocals of each other.

5. Derivative Rules

→ constant, power, product, quotient

6. Chain Rule

→ used for composite functions

7. **<u>Test for differentiability</u>**:

$(i)\ f(a)\ exist$

$$(ii)\ f'(a) = \frac{f(x) - f(a)}{x - a}\ must\ be\ defined$$

8. <u>Higher order Derivatives</u>

$f(x)$ → position function

$f'(x)$ → velocity function

$f''(x)$ → acceleration function

9. <u>Implicit Differentiation</u>

→ determine the derivative of an implicit relation