

MEMORIAL UNIVERSITY OF NEWFOUNDLAND
DEPARTMENT OF MATHEMATICS AND STATISTICS

SAMPLE

Calculus Placement Test

COMPLETE THE FOLLOWING CAREFULLY AND CLEARLY:

(Please Print)

Surname: _____

Given Names: _____

MUN Number: _____

Please note:

This exam has **EIGHT** pages of questions.

All calculators are strictly forbidden.

The questions are to be answered in the spaces provided.

Under no circumstances may the candidate take this book from the examination room.

On no account are pages to be torn or removed from this book, unless specifically directed.

Candidates must not have in their possession books, notes or papers of any kind, unless specifically directed.

No electronic devices of any kind, including cell phones and MP3 players, are permitted at your desk.

	MARKS
16	1. _____
25	2. _____
9	3. _____
11	4. _____
7	5. _____
7	6. _____
18	7. _____
7	8. _____
100	Total _____

In the multiple choice questions, #1 and #2, there is only one correct answer, but the other answers have been chosen to reflect a specific error made in the calculations to arrive at the answer. Students may be given part marks depending on which of the incorrect answers they choose.

1. Evaluate each of the following limits, or indicate that the limit does not exist (assigning ∞ or $-\infty$ where appropriate), by checking one of the corresponding boxes. (You may check only one box per question. If more than one box is checked, you will not receive any credit for that question.)

[4] (a) $\lim_{x \rightarrow -5} \frac{x^2 - 25}{x^3 + 5x^2 + 5x + 25}$

1

-1

$-\frac{1}{3}$

$\frac{1}{5}$

does not exist

[4] (b) $\lim_{x \rightarrow 2} \frac{-3 + \sqrt{4x + 1}}{10 - 5x}$

$-\frac{2}{15}$

$-\frac{4}{5}$

$-\frac{1}{5}$

$\frac{2}{3}$

does not exist

[4] (c) $\lim_{x \rightarrow 3^+} \frac{|3 - x|}{x^2 - 6x + 9}$

0

$-\frac{1}{9}$

∞

$-\infty$

does not exist

[4] (d) $\lim_{x \rightarrow -\infty} \frac{2x - 1}{x - \sqrt{9x^2 + 7}}$

-1

$\frac{1}{2}$

0

2

does not exist

2. Compute the derivative of y with respect to x , by checking one of the corresponding boxes. (You may check only one box per question. If more than one box is checked, you will not receive any credit for that question.)

[5] (a) $y = \csc(x^2 e^x)$

$-\csc(x^2 e^x) \cot(x^2 e^x)$

$-\csc^2(x^2 e^x)$

$-x e^x (x + 2) \csc(x^2 e^x) \cot(x^2 e^x)$

$-x e^x (x + 2) \csc^2(x^2 e^x)$

$-2x e^x \csc(x^2 e^x) \cot(x^2 e^x)$

$-2x e^x \csc^2(x^2 e^x)$

[5] (b) $y = \frac{\arctan(x)}{1+x^2}$

$\frac{1}{2x(1+x^2)}$

$\frac{\sec^2(x)}{2x}$

$\frac{1-2x\arctan(x)}{(1+x^2)^2}$

$\frac{2x\arctan(x)-1}{(1+x^2)^2}$

$\frac{1-2x\arctan(x)}{1+x^2}$

$\frac{2x\arctan(x)-1}{1+x^2}$

[5] (c) $y = \sin(e^{x^4})$

$\cos(e^{x^4})$

$4x^3 \cos(e^{x^4})$

$\cos(4x^3 e^{x^4})$

$4x^3 e^{x^4} \cos(e^{x^4})$

$e^{x^4} \cos(e^{x^4})$

$4x^3 e^{x^4} \sin(e^{x^4}) \cos(e^{x^4})$

[5] (d) $y = x^{\sin(x)}$

$x^{\sin(x)-1}$

$\cos(x) \cdot x^{\sin(x)-1}$

$x^{\sin(x)} \ln(\sin(x))$

$x^{\sin(x)} \cdot \frac{\cos(x)}{x}$

$x^{\sin(x)} [\cos(x) \ln(x) + 1]$

$x^{\sin(x)} \left[\cos(x) \ln(x) + \frac{\sin(x)}{x} \right]$

[5] (e) $xy^2 = 2x - y$

$\frac{2-y^2}{2xy+1}$

$\frac{2}{2xy+1}$

$\frac{2-y^2}{2x+1}$

$\frac{2}{2x+1}$

$\frac{2}{y^2} - \frac{1}{xy}$

$-\frac{4}{y^3} + \frac{1}{x^2y}$

3. Consider the function

$$f(x) = \begin{cases} \frac{4x + 8}{x^2 - 2x - 8}, & \text{for } x \leq 0 \\ x^2 - 2x, & \text{for } x > 0. \end{cases}$$

- [4] (a) Use the definition of continuity to determine whether $f(x)$ is continuous at $x = 0$. If it is not, is the discontinuity removable or non-removable?

- [5] (b) Use the definition of continuity to determine all other points at which $f(x)$ is not continuous. Classify any discontinuities as removable or non-removable.

[7] 4. (a) Use the definition of the derivative to differentiate $f(x) = \frac{2x}{5-x}$.

[4] (b) Find the equation of the tangent line to the curve $y = \frac{2x}{5-x}$ at the point $x = 4$.

- [7] 5. A spotlight on the ground shines on a wall 12 metres away. If a man 2 metres tall walks from the spotlight toward the building at a speed of 1 metre per second, how fast is his shadow on the building shrinking when he is 4 metres from the building?

- [7] 6. A box is to be manufactured such that its width is exactly three times its length and its volume is 36 cubic metres. Assuming that the box is closed on all sides, determine the smallest amount of material needed for its construction.

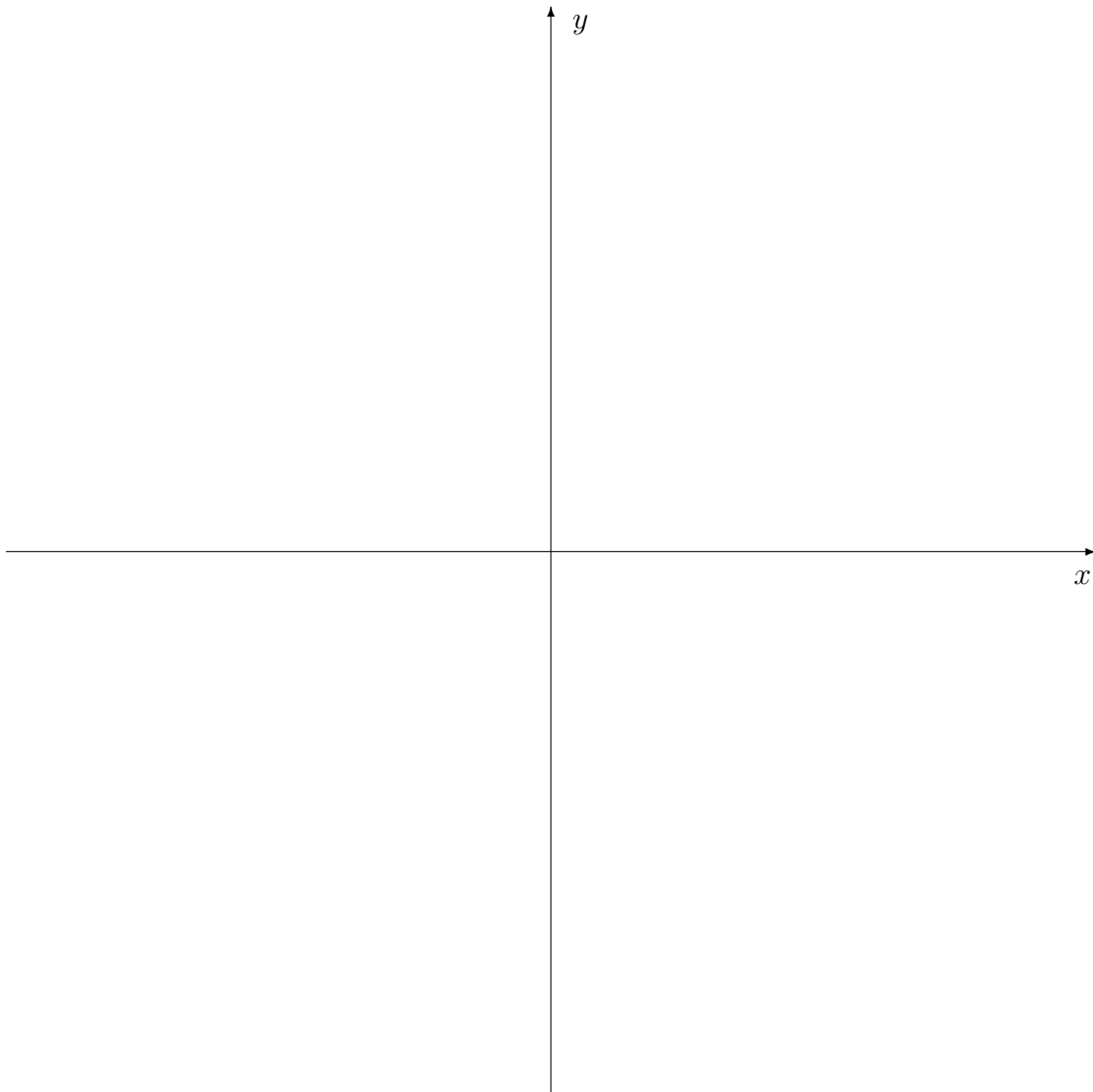
7. Consider the function $f(x) = \frac{9(x^2 - 16)}{(x + 8)^2}$ with derivatives

$$f'(x) = \frac{144(x + 2)}{(x + 8)^3} \quad \text{and} \quad f''(x) = \frac{-288(x - 1)}{(x + 8)^4}.$$

- [3] (a) Find the vertical asymptotes of the graph of $f(x)$, if any.
- [3] (b) Find the horizontal asymptotes of the graph of $f(x)$, if any.
- [2] (c) Find the x - and y -intercepts of the graph of $f(x)$, if any.
- [3] (d) Determine the intervals on which $f(x)$ is increasing or decreasing, and classify any relative (local) extrema.

- [3] (e) Determine the intervals on which $f(x)$ is concave upward or concave downward, and identify any points of inflection.

- [4] (f) Sketch the graph of $f(x)$ on the axes provided. Label your graph carefully.



- [7] 8. Use the definition of the derivative to prove that if $f(x)$ and $g(x)$ are differentiable functions then

$$[f(x)g(x)]' = f'(x)g(x) + g'(x)f(x).$$