Mathematics 3208

Name:__________

Worksheet: Calculus of Exponential and Logarithmic Functions

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1. Determine the derivative for each of the following and make any obvious simplifications.

a) $f(x) = x^4 e^{\sin(2x)}$

$f'(x) = x^4 e^{\sin(2x)} \cos(2x) \cdot 2 + e^{\sin 2x} (4x^3)$

$f'(x) = 2x^4 e^{\sin(2x)} \cos(2x) + 4e^{\sin 2x}(x^3)$

$f'(x) = 2x^3 e^{\sin 2x} [x \cos(2x) + 2]$

b) $f(x) = \csc(x^3 e^x)$

$f'(x) = -\csc(x^3 e^x) \cot(x^3 e^x) [x^3 e^x + e^x (3x^2)]$

$f'(x) = -x^2 e^x [x+3] \csc(x^3 e^x) \cot(x^3 e^x)$

c) $f(x) = \sqrt{\sec(6^x)} = [\sec(6^x)]^{\frac{1}{2}}$

$f'(x) = \frac{1}{2}[\sec(6^x)]^{-\frac{1}{2}} [\sec(6^x)\tan(6^x)] \, 6^x (\ln 6)$

$f'(x) = \dfrac{6^x \ln 6 \sec(6^x) \tan(6^x)}{2\sqrt{\sec(6^x)}}$

d) $f(x) = 10^{\cos(7x)}$

$$f'(x) = 10^{\cos 7x} (\ln 10) (-\sin 7x) \quad (7)$$

$$f'(x) = -7 \ln 10 \sin(7x) \; 10^{\cos 7x}$$

e) $f(x) = \left(\dfrac{e^{2x}-1}{e^{2x}+1}\right)^4$

$$f'(x) = 4\left(\frac{e^{2x}-1}{e^{2x}+1}\right)^3 \left[\frac{(e^{2x}+1)(e^{2x})(2) - (e^{2x}-1)(e^{2x})(2)}{(e^{2x}+1)^2}\right]$$

$$f'(x) = 4\left(\frac{e^{2x}-1}{e^{2x}+1}\right)^3 \left[\frac{2e^{4x} + 2e^{2x} - 2e^{4x} + 2e^{2x}}{(e^{2x}+1)^2}\right]$$

$$f'(x) = 4\left(\frac{e^{2x}-1}{e^{2x}+1}\right)^3 \left[\frac{4e^{2x}}{(e^{2x}+1)^2}\right]$$

$$f'(x) = \frac{16 e^{2x} (e^{2x}-1)^3}{(e^{2x}+1)^5}$$

f) $f(x) = \ln\left[(2x+7)^7 (3x+9)^9\right]$

$$f(x) = \ln(2x+7)^7 + \ln(3x+9)^9$$

$$f(x) = 7\ln(2x+7) + 9\ln(3x+9)$$

$$f'(x) = 7\left(\frac{1}{2x+7}\right)(2) + 9\left(\frac{1}{3x+9}\right)(3)$$

$$f'(x) = \frac{14}{2x+7} + \frac{27}{3x+9}$$

g) $f(x) = e^{\frac{x}{4-x^2}}$

$$f'(x) = e^{\frac{x}{4-x^2}} \cdot \left[\frac{(4-x^2)(1) - (x)(-2x)}{(4-x^2)^2} \right]$$

$$f'(x) = e^{\frac{x}{4-x^2}} \left[\frac{4 - x^2 + 2x^2}{(4-x^2)^2} \right]$$

$$f'(x) = e^{\frac{x}{4-x^2}} \left[\frac{4 + x^2}{(4-x^2)^2} \right]$$

h) $f(x) = 5^{\tan x} \ln(\sec(4x))$

$$f'(x) = 5^{\tan x} \cdot \frac{1}{\sec(4x)} \sec(4x) \tan(4x)(4) + \ln(\sec 4x) \cdot 5^{\tan x} (\ln 5)(\sec^2 x)$$

$$f'(x) = 4(5)^{\tan x} \tan(4x) + \ln 5 \, (5^{\tan x}) \ln(\sec 4x) \sec^2 x$$

i) $f(x) = \cos(e^{x^4})$

$$f'(x) = -\sin(e^{x^4}) \cdot e^{x^4} (4x^3)$$

$$f'(x) = -4x^3 e^{x^4} \sin(e^{x^4})$$

j) $f(x) = \ln\left(\frac{3^x+1}{3^x-1}\right)^5$

$f(x) = 5\ln\left(\frac{3^x+1}{3^x-1}\right)$

$f(x) = 5\left[\ln(3^x+1) - \ln(3^x-1)\right]$

$f'(x) = 5\left[\frac{1}{3^x+1} \cdot 3^x \ln 3 - \frac{1}{3^x-1}\ 3^x \ln 3\right]$

$f'(x) = 5(\ln 3)(3^x)\left[\frac{1}{3^x+1} - \frac{1}{3^x-1}\right]$

k) $y = \log_4(\sin(e^{2x}))$

$y' = \frac{1}{\sin(e^{2x})} \cdot \frac{1}{\ln 4} \cdot \cos(e^{2x}) \cdot e^{2x}\ (2)$

$y' = \frac{2e^{2x}\cos(e^{2x})}{\ln 4\ \sin(e^{2x})}$

l) $y = 5^{x^3}\log(x^4)$

$y' = 5^{x^3} \cdot \frac{1}{x^4 \ln 10}\ (4x^3) + \log(x^4) \cdot 5^{x^3}\ln(5)\ (3x^2)$

$y' = \frac{4(5^{x^3})}{x\ln 10} + (3x^2)\ln(5)\ 5^{x^3}\ \log(x^4)$

$y' = 5^{x^3}\left[\frac{4}{x\ln 10} + 3x^2(\ln 5)\ \log(x^4)\right]$

m) $y = \log_3 \sqrt{x^5 + x^2}$

$y = \log_3 (x^5 + x^2)^{\frac{1}{2}}$

$y = \frac{1}{2} \log_3 (x^5 + x^2)$

$y' = \frac{1}{2} \left[\frac{1}{(x^5+x^2)\ln 3} (5x^4 + 2x) \right]$

$y' = \frac{5x^4 + 2x}{2\ln 3 (x^5 + x^2)}$

2. Determine $\frac{dy}{dx}$ for each of the following:

a) $y = \frac{(2x-1)^4}{(x^3+1)^5 \sqrt{8x^2+1}}$

$\ln y = \ln \left[\frac{(2x-1)^4}{(x^3+1)^5 (8x^2+1)^{\frac{1}{2}}} \right]$

$\ln y = \ln (2x-1)^4 - \ln (x^3+1)^5 - \ln (8x^2+1)^{\frac{1}{2}}$

$\ln y = 4 \ln (2x-1) - 5 \ln (x^3+1) - \frac{1}{2} \ln (8x^2+1)$

$\frac{1}{y} \cdot \frac{dy}{dx} = 4\left(\frac{1}{2x-1}\right) 2 - 5\left(\frac{1}{x^3+1}\right)(3x^2) - \frac{1}{2} \cdot \left(\frac{1}{8x^2+1}\right)(16x)$

$\frac{1}{y} \cdot \frac{dy}{dx} = \frac{8}{2x-1} - \frac{15x^2}{x^3+1} - \frac{8x}{8x^2+1}$

$\frac{dy}{dx} = \frac{(2x-1)^4}{(x^3+1)^5 \sqrt{8x^2+1}} \left[\frac{8}{2x-1} - \frac{15x^2}{x^3+1} - \frac{8x}{8x^2+1} \right]$

b) $y = x^{\cos x}$

$\ln y = \ln (x^{\cos x})$

$\ln y = (\cos x)(\ln x)$

$\frac{1}{y} \cdot \frac{dy}{dx} = \cos x \left(\frac{1}{x}\right) + \ln x\,(-\sin x)$

$\frac{1}{y} \frac{dy}{dx} = \frac{\cos x}{x} - (\ln x)(\sin x)$

$\frac{dy}{dx} = x^{\cos x}\left[\frac{\cos x}{x} - (\ln x)(\sin x)\right]$

c) $y = (\sin x)^{\sin x}$

$\ln y = \ln (\sin x)^{\sin x}$

$\ln y = (\sin x) \ln(\sin x)$

$\frac{1}{y} \cdot \frac{dy}{dx} = (\sin x) \cdot \frac{1}{\sin x} (\cos x) + \ln(\sin x) \cdot (\cos x)$

$\frac{1}{y} \cdot \frac{dy}{dx} = \cos x + \cos x \ln(\sin x)$

$\frac{dy}{dx} = (\sin x)^{\sin x}\left[\cos x + \cos x \ln(\sin x)\right]$

$\frac{dy}{dx} = (\sin x)^{\sin x} \cdot \cos x \left[1 + \ln(\sin x)\right]$

3. Find the equation of the tangent line to the curve $2x - y\ln y = 4$ at the point $(2,1)$.

$x=2$
$y=1$

$$2x - y\ln y = 4$$
$$2 - \left[y \cdot \frac{1}{y} \cdot \frac{dy}{dx} + \ln y \cdot \frac{dy}{dx}\right] = 0$$
$$2 - \frac{dy}{dx} - \ln y \frac{dy}{dx} = 0$$
$$2 = \frac{dy}{dx} + \ln y \frac{dy}{dx}$$
$$2 = \frac{dy}{dx}(1 + \ln y)$$
$$\frac{2}{1+\ln y} = \frac{dy}{dx}$$

$$y - 1 = 2(x-2)$$

$$\frac{2}{1+\ln(1)} = \frac{dy}{dx}$$
$$2 = \frac{dy}{dx} \quad m_{tangent}$$

4. Find the equation of the normal line to the curve $y = \ln(e^x + e^{2x})$ at the point $(0, \ln 2)$

$\left.\begin{matrix} x=0 \\ y=\ln 2 \end{matrix}\right\}$

$$y = \ln(e^x + e^{2x})$$
$$y' = \frac{1}{e^x + e^{2x}}\left[e^x + e^{2x}(2)\right]$$
$$y' = \frac{e^x + 2e^{2x}}{e^x + e^{2x}}$$
$$y' = \frac{e^0 + 2e^0}{e^0 + e^0} = \frac{1+2}{2} = \frac{3}{2}$$

$$m_{tangent} = 3/2$$

$$m_{normal} = -2/3$$

$$y - \ln 2 = -2/3\,(x - 0)$$