Lesson 10.2: Products and Quotients of Functions

To combine two functions, $f(x)$ and $g(x)$, simply multiply or divide or as follows:

Quotient

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$$

Product

$$(f \bullet g)(x) = f(x)g(x)$$

Products of Functions

→ Multiplying the y-values of two functions, at corresponding values of x, produces a new function.

Algebraically

$f(x) = x + 3$

$g(x) = 2x - 1$

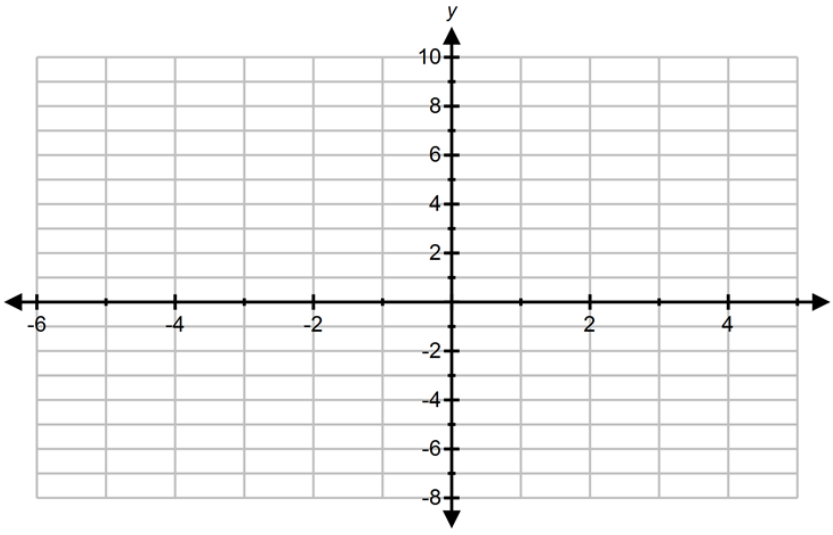
$h(x) = f(x)g(x)$

$h(x) =$

Numerically

x	$f(x)$	$g(x)$	$h(x)$
-4			
-3			
-1			
0			
1			
2			

Graphically

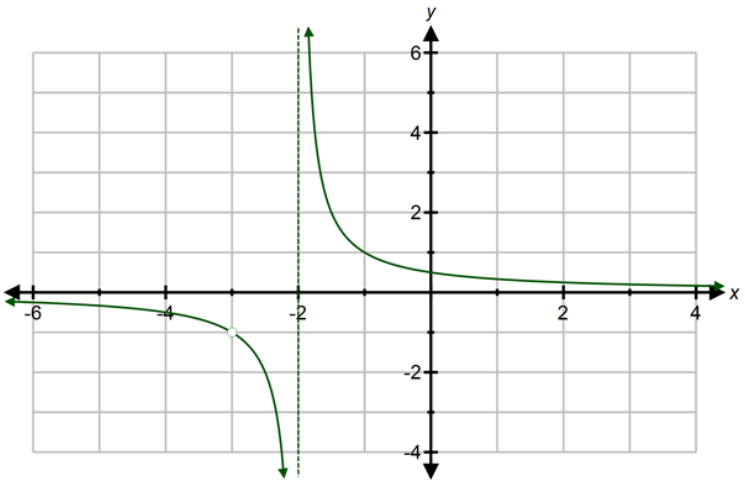


Quotients of Functions

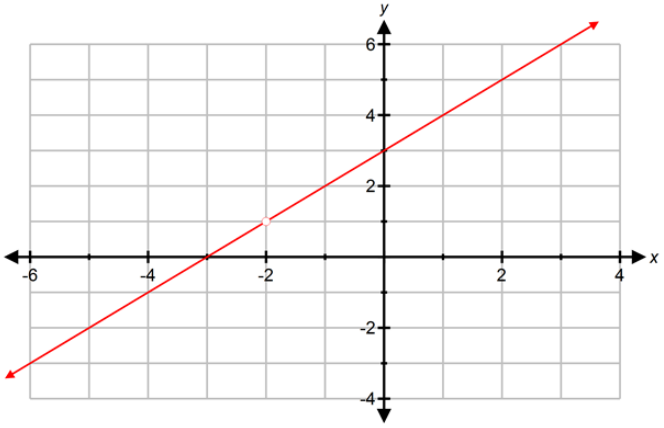
Non-Permissible Values

The domain of a quotient $h(x) = \frac{f(x)}{g(x)}$ is restricted for values of x where $g(x) = 0$.

Non-Permissible Values → vertical asymptote
→ point of discontinuity



$$y = \frac{x+3}{x^2+5x+6}$$



$$y = \frac{x^2+5x+6}{x+2}$$

Example 2

(i) Use the functions $f(x) = 2x + 8$ and $g(x) = x^2 + 3x - 4$ to determine $\left(\frac{f}{g}\right)(x)$ and state the non-permissible values.

(ii) How would you determine the value $\left(\frac{f}{g}\right)(5)$?

Your Turn

Use the functions $f(x) = x + 2$ and $g(x) = x^2 + 4x + 3$ to determine $\left(\frac{f}{g}\right)(x)$ and state the non-permissible values.

Quotients of Functions

→ Dividing the y-values of two functions, at corresponding values of x, also produces a new function.

Algebraically

$f(x) = x^2 + x - 6$

$g(x) = x - 2$

$h(x) = \dfrac{f(x)}{g(x)}$

$h(x) =$

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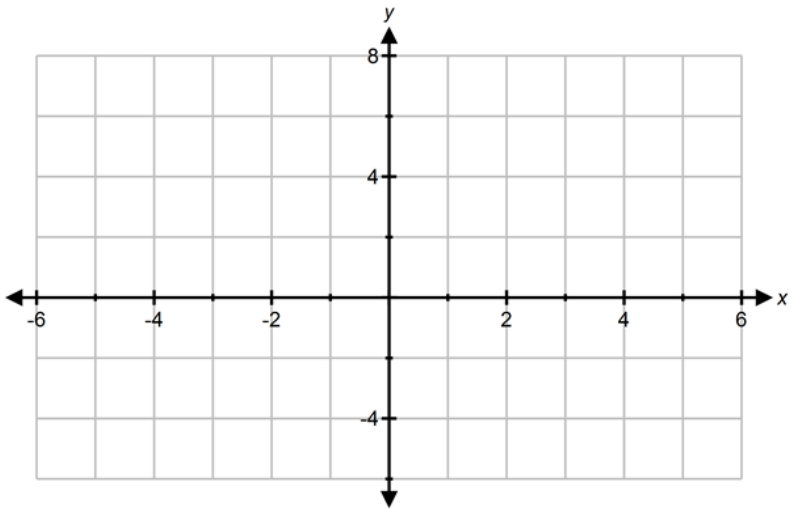
Numerically

x	$f(x)$	$g(x)$	$h(x)$
-2			
-1			
0			
1			
2			

Graphically



Domain

$f \rightarrow$

$g \rightarrow$

$h \rightarrow$

Example 3

Consider the functions $f(x) = x^2 + 3x + 2$ and $g(x) = 2x^2 + 3x - 2$

(i) Algebraically determine an equation for $h(x) = \left(\frac{f}{g}\right)(x)$

(ii) State the domain of $h(x)$.

Equation

Domain

Your Turn

Consider the functions $f(x) = 2x^2 + 7x - 15$ and $g(x) = 3x^2 + 16x + 5$

(i) Algebraically determine an equation for $h(x) = \left(\frac{f}{g}\right)(x)$

(ii) State the domain $h(x)$.

Equation

Domain

Your Turn

Given $f(x) = 3x - 1$ and $g(x) = 3x^2 + 5x - 2$, what is the domain of $h(x) = \left(\frac{f}{g}\right)(x)$?

A) $\{x \mid x \in R\}$

B) $\left\{x \mid x \neq \frac{1}{3}, x \in R\right\}$

C) $\{x \mid x \neq -2, x \in R\}$

D) $\left\{x \mid x \neq -2, x \neq \frac{1}{3}, x \in R\right\}$