

## Lesson 10.3: Composite Functions

↳ Two functions  $f(x)$  and  $g(x)$  can be combined using **composition** to produce new functions .

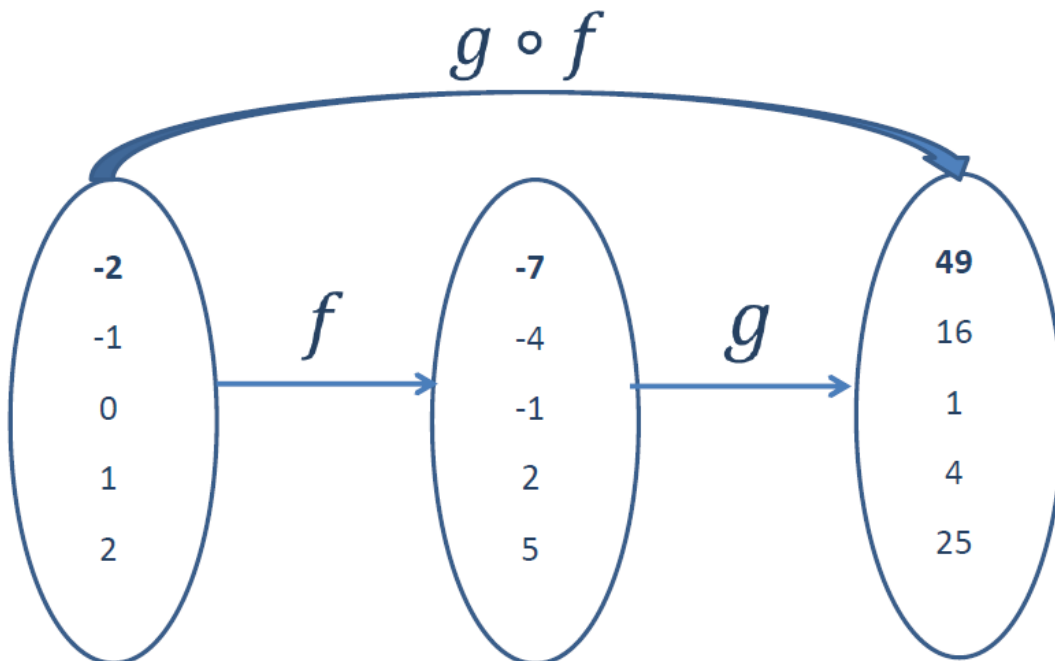
<p>Composite Function</p> $(f \circ g)(x) = f(g(x))$ $(g \circ f)(x) = g(f(x))$
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**Careful:**  
 $f(g(x))$  is not the product of the two functions

## Composite Functions

↳ Consider the functions  $f(x) = 3x - 1$  and  $g(x) = x^2$  .

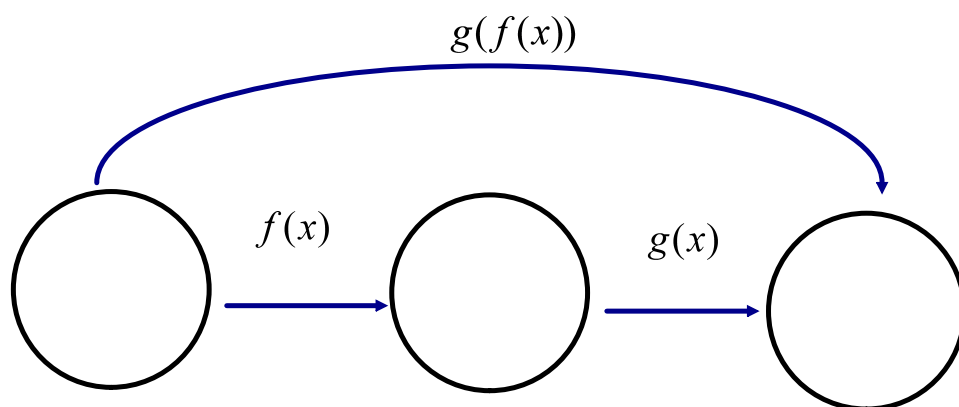
We can compose these to create  $(g \circ f)(x)$



## Compose Algebraically

Consider the functions  $f(x) = 3x - 1$  and  $g(x) = x^2$ .

We can compose these to create  $(g \circ f)(x)$



→

## Lesson 10.3 Composite Functions

### Example 1

Given  $f(x) = 2x + 1$ ,  $g(x) = x^2 - 2x + 3$  determine the following :

i)  $f(g(1))$

ii)  $g(f(-2))$

iii)  $f(f(0))$

### Example 2

Let  $f(x) = \sqrt{x-2}$  and  $g(x) = 4x^2 + 1$ . Evaluate the following:

(i)  $f(g(3))$

(ii)  $f(g(-1))$

(iii)  $f(g(0))$

(iv)  $g(f(2))$



## Lesson 10.3 Composite Functions

### Example 3

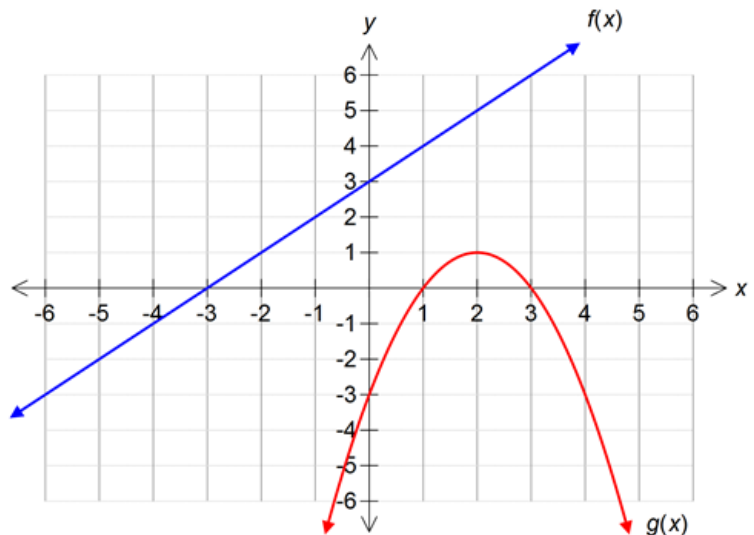
Use the graphs of  $f(x)$  and  $g(x)$  to evaluate the following:

(i)  $f(g(1))$

(ii)  $f(g(2))$

(iii)  $g(f(1))$

(iv)  $g(f(-3))$



### Example 4

Let  $f(x) = x + 1$  and  $g(x) = x^2$ . Determine the following:

(a)  $f(g(x))$

(b)  $g(f(x))$

(c)  $f(f(x))$

(d)  $g(g(x))$

→

## Lesson 10.3 Composite Functions

### Example 5

Given  $f(x) = 2x^2 - 3x + 1$  and  $g(x) = 5x - 3$ , determine the simplified equations for

i)  $(f \circ g)(x)$

ii)  $(g \circ f)(x)$



## Domain of the Composition of Two Functions

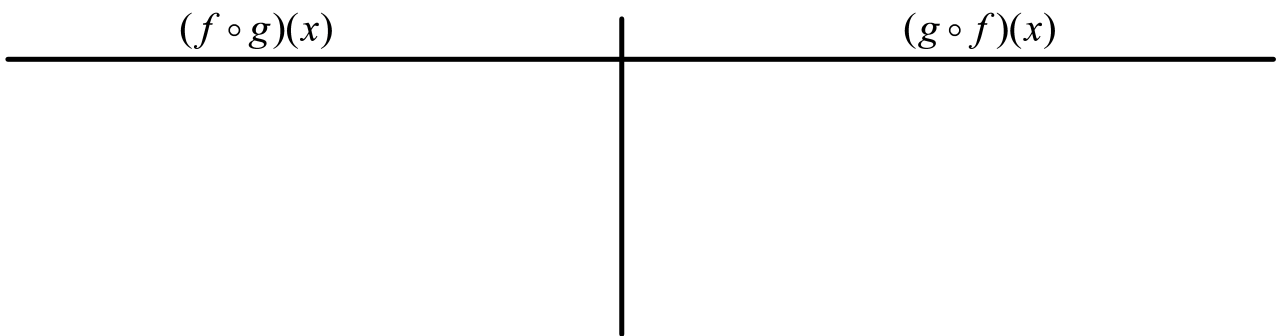
└ identify the restriction on the inner function and the composite function

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### Example 6

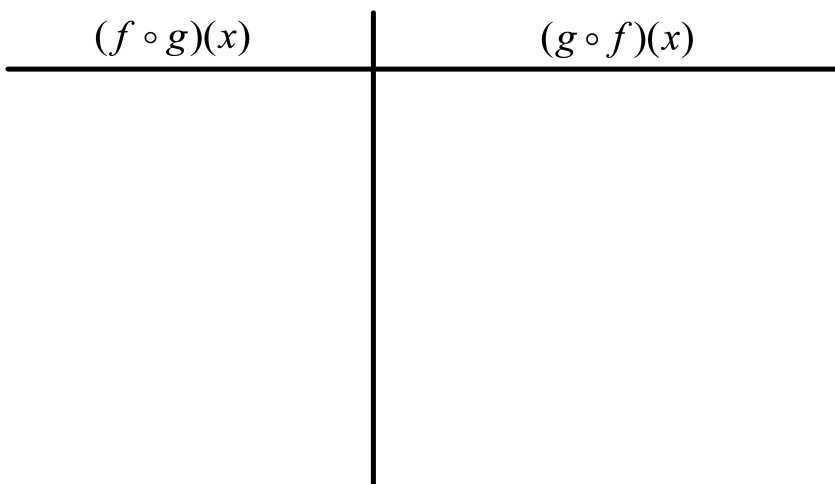
Let  $f(x) = \sqrt{x}$  and  $g(x) = 4x + 1$ .

- i) Algebraically determine  $(f \circ g)(x)$  and  $(g \circ f)(x)$ .
- ii) Determine the domains of  $f \circ g$  and  $g \circ f$ .



### Example 7

Given  $f(x) = \sqrt{x}$  and  $g(x) = x^2$ , what are the domains of  $(f \circ g)(x)$  and  $(g \circ f)(x)$ .



- | Domain of<br>$(f \circ g)(x)$ | Domain of<br>$(g \circ f)(x)$ |
|-------------------------------|-------------------------------|
| A) $x \in R$                  | $x \geq 0$                    |
| B) $x \geq 0$                 | $x \geq 0$                    |
| C) $x \in R$                  | $x \in R$                     |
| D) $x \geq 0$                 | $x \in R$                     |

## Lesson 10.3 Composite Functions

### Example 8

Let  $f(x) = x + 2$  and  $g(x) = \frac{1}{x}$ . Determine  $g(f(x))$  and  $f(g(x))$  and state the domains.

### Example 9

Let  $g(x) = \frac{1}{x}$  and  $f(x) = \sqrt{x - 3}$ . Determine the domain of  $(g \circ f)(x)$ .

→

## Write the Equation of the Original Functions Given the Composite Function

### Example 10

If  $h(x) = f(g(x))$  determine the original functions  $f(x)$  and  $g(x)$  where  $h(x) = (x-1)^2 + (x-1) + 3$ .

### Example 11

If  $h(x) = f(g(x))$ , determine the original functions  $f(x)$  and  $g(x)$  where  $h(x) = (x+5)^2$ . State two possible answers.

