

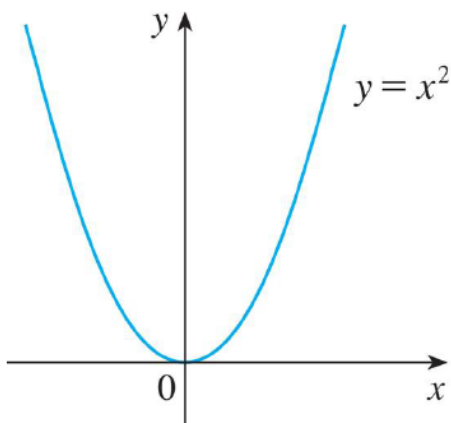
Lesson 2.10 Types of Discontinuities

1. Determine all discontinuities for a given function
2. Characterize all discontinuities using the definition of continuity.

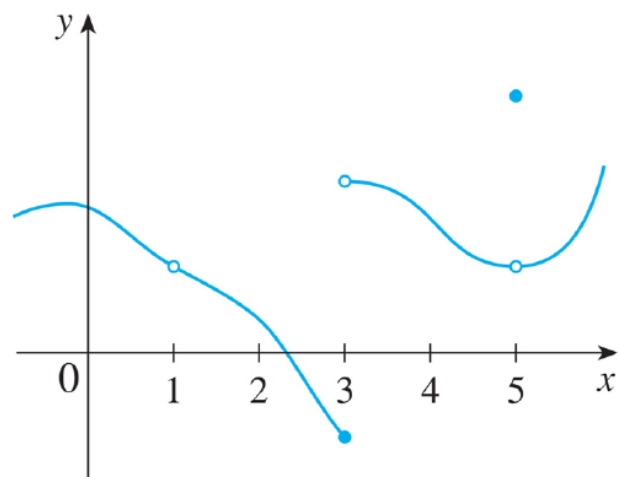
Continuous Function

- A function is continuous if there are no holes, breaks or jumps in its graph.
- It is often said that a function is continuous if you can draw its graph *"without lifting your pencil"* from the paper.

Continuous Function



Discontinuous Function



Defining Continuity at a Point

↳ A function $f(x)$ is continuous at the point where $x = a$ if

$$\lim_{x \rightarrow a} f(x) = f(a)$$

Three criteria:

1) the limit exists $\longrightarrow \lim_{x \rightarrow a^+} f(x) = \lim_{x \rightarrow a^-} f(x)$

2) the function exists $\longrightarrow f(a)$ has a value

3) the function equals the limit $\longrightarrow \lim_{x \rightarrow a} f(x) = f(a)$

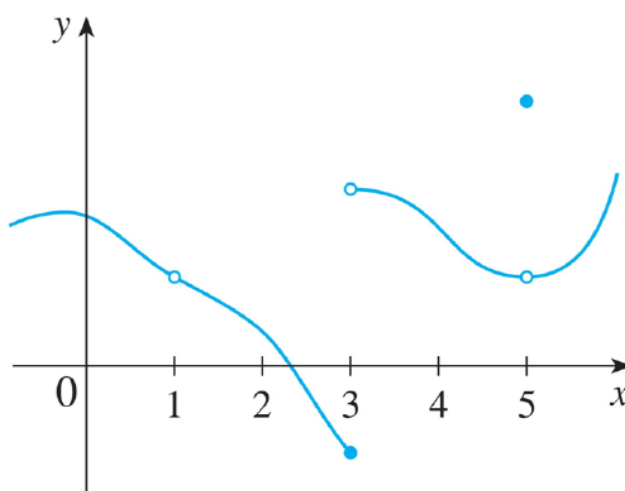
Example 1

Understanding the Definition

i) $x = 1$

$$\hookrightarrow \lim_{x \rightarrow 1^+} f(x) = \lim_{x \rightarrow 1^-} f(x)$$

$f(1) = \text{undefined}$



ii) $x = 3$

$$\hookrightarrow \lim_{x \rightarrow 3^+} f(x) \neq \lim_{x \rightarrow 3^-} f(x)$$

iii) $x = 5$

$$\hookrightarrow \lim_{x \rightarrow 5^+} f(x) = \lim_{x \rightarrow 5^-} f(x)$$

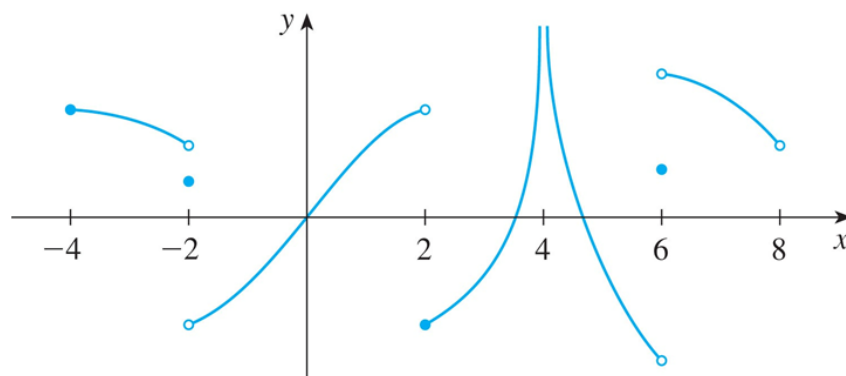
$f(5)$ exists

$$\lim_{x \rightarrow 5} f(x) \neq f(5)$$

Lesson 2.10 Types of Discontinuities

Your Turn

List values of x at which the function whose graph is shown is discontinuous. At each of these points give a reason why the function violates the definition of continuity.



Lesson 2.10 Types of Discontinuities

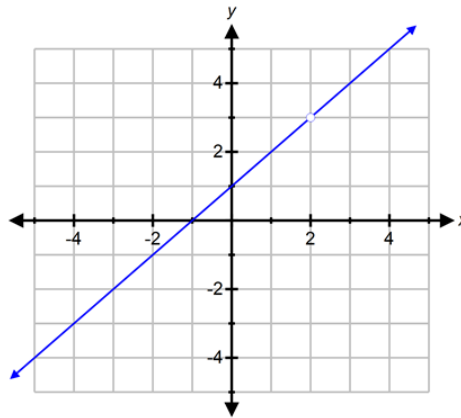
Types of discontinuities

- (i) removable
 - (ii) jump
 - (iii) infinite
- } non-removable

At a particular point we can classify three types of discontinuities. Each category is based on the way in which the functions violates the definition of the continuity at that point.

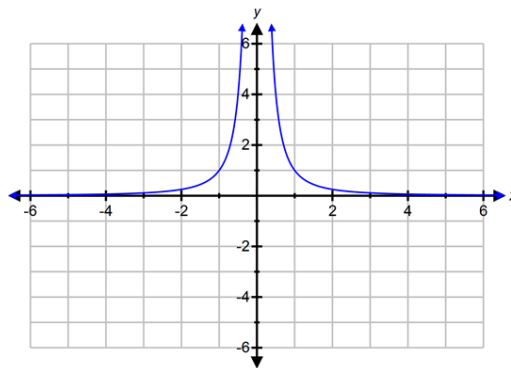
Removable Discontinuity

$$f(x) = \frac{x^2 - x - 2}{x - 2}$$

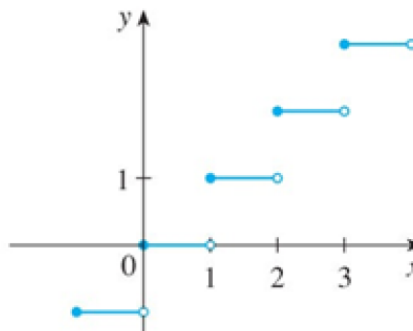


Infinite Discontinuity

$$f(x) = \frac{1}{x^2}$$



Jump Discontinuity



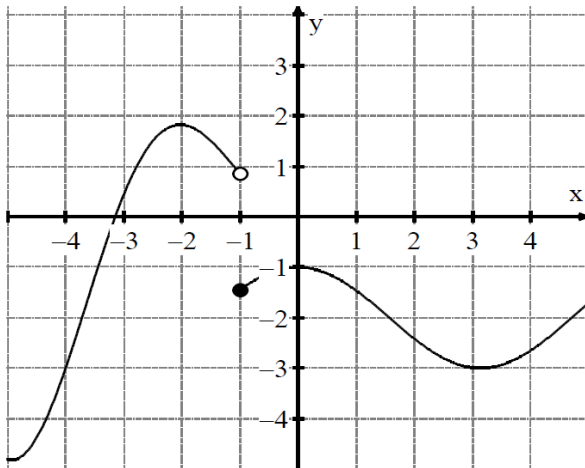
(d) $f(x) = \llbracket x \rrbracket$

Lesson 2.10 Types of Discontinuities

Your Turn

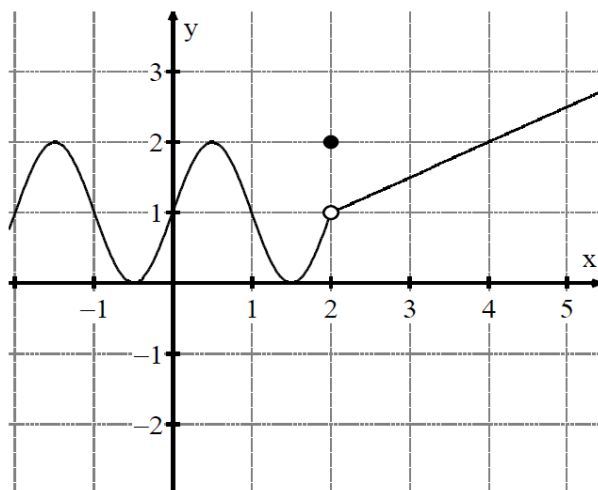
What type of discontinuity is illustrated by the graphs of $f(x)$ as shown.

(i)



- A) Finite Discontinuity
- B) Infinite Discontinuity
- C) Jump Discontinuity
- D) Removable Discontinuity

(ii)



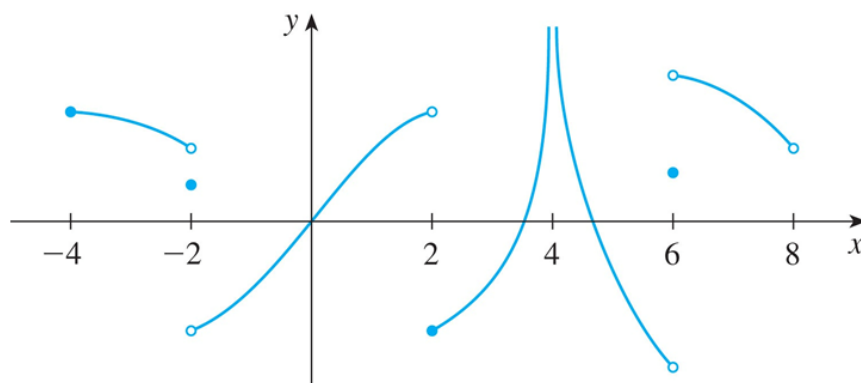
- A) Finite Discontinuity
- B) Infinite Discontinuity
- C) Jump Discontinuity
- D) Removable Discontinuity



Lesson 2.10 Types of Discontinuities

Your Turn

List values of x at which the function whose graph is shown is discontinuous. At each of these points, state the type of discontinuity illustrated.



Key Ideas

- Discontinuities can be classified as; removable, jump, or infinite.
- For **removable discontinuities** $\lim_{x \rightarrow a^-} f(x) = \lim_{x \rightarrow a^+} f(x)$
so the limit exists, but $\lim_{x \rightarrow a} f(x) \neq f(a)$
- For **infinite discontinuities**, there is a vertical asymptote.
- For jump discontinuities, $\lim_{x \rightarrow a^-} f(x) \neq \lim_{x \rightarrow a^+} f(x)$

