Definition of Continuity

Three conditions necessary for a function to be continuous at a particular value of x :

A function is continuous at $x = a$ when:

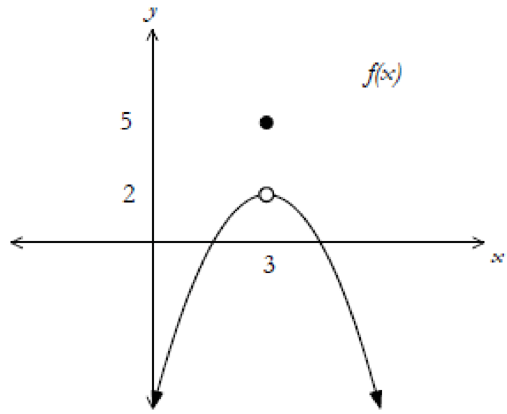
(1) $f(a)$ is defined

(2) $\lim_{x \to a} f(x)$ exists

(3) $\lim_{x \to a} f(x) = f(a)$

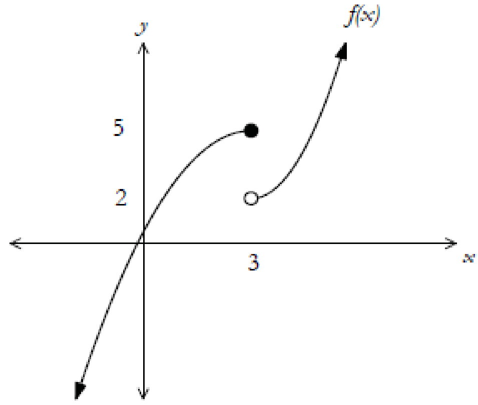
Example 1

Is the following function continuous at x = 3?



Example 2

Is the following function continuous at x = 3?



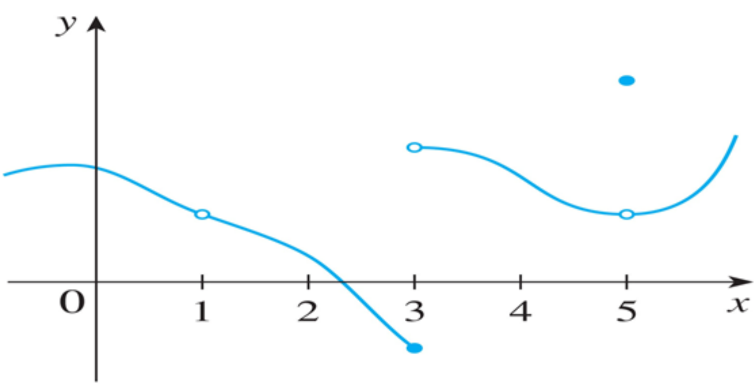
Continuity from Left and Right

A function may have a point of discontinuity but still be considered continuous from either the left or the right.

- A function f is continuous from the right at a number a if $\lim_{x \to a^+} f(x) = f(a)$
- A function f is continuous from the left at a number a if $\lim_{x \to a^-} f(x) = f(a)$

$\lim_{x \to 3^+} f(x) =$

$\lim_{x \to 3^-} f(x) =$



Example 3

Using the definition of continuity, identify all points at which the function is discontinuous. Classify these discontinuities as removable or non removable.

$$f(x)=\begin{cases}3x-2, x<3\\ x^2+x-8, x\geq 3\end{cases}$$

Example 4

Using the definition of continuity, identify all points at which the function is discontinuous. Classify these discontinuities as removable or non removable.

$$f(x)=\begin{cases}\dfrac{3x+12}{x^2+x-12}, & x\le 0\\[2ex] \dfrac{x+4}{x-4}, & x>0\end{cases}$$

Example 5

Using the definition of continuity, identify all points at which the function is discontinuous. Classify these discontinuities as removable or non removable.

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$$f(x)=\begin{cases}\dfrac{4x+8}{x^2-2x-8},\ for\ x\le 0\\[2ex] x^2-2x,\ for\ x>0\end{cases}$$

Example 6

Determine the value of *a* and *b* so that the function is continuous?

$$f(x)=\begin{cases} 2, x \le -1 \\ ax+b, -1 < x < 3 \\ -2, x \ge 3 \end{cases}$$