

## Lesson 2.6: Factoring to Evaluate Limits

↳ There are times when determining the  $\lim_{x \rightarrow a} \frac{f(x)}{g(x)}$  substitution results in the form  $\frac{0}{0}$ .  
↙ **Indeterminate Form**

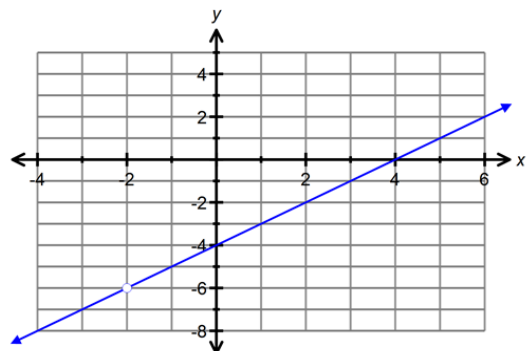
↳ To deal with indeterminate forms caused by zero denominators, a factoring strategy can sometimes help.

### Example 1

Evaluate  $\lim_{x \rightarrow -2} \frac{x^2 - 2x - 8}{x + 2}$

Algebra

Graph



Why is it okay to factor and cancel? Doesn't this ignore the non-permissible value?

↳ Remember the definition of a limit.

$$\lim_{x \rightarrow a} f(x) = L$$

**if and only if**

$$\lim_{x \rightarrow a^-} f(x) = \lim_{x \rightarrow a^+} f(x) = L$$

## Lesson 2.6 Evaluate limits Indeterminate Form

### Example 2

Evaluate  $\lim_{x \rightarrow 3} \frac{x^2 - x - 6}{x - 3}$

### Example 3

Evaluate  $\lim_{x \rightarrow 4} \frac{x^3 - 4x^2 - 9x + 36}{2x^2 - 7x - 4}$

### Example 4

Evaluate  $\lim_{x \rightarrow 3} \frac{x^3 - 5x^2 + 6x}{x^4 - 81}$

Example 5

Evaluate  $\lim_{x \rightarrow -2} \frac{x + 2}{x^3 + 8}$



Example 6

Evaluate  $\lim_{x \rightarrow 10} \frac{(x - 5)^2 - 25}{x - 10}$

## Indeterminate Forms

What are some other types of algebraic techniques that can be applied to simplify the expression  $\frac{f(x)}{g(x)}$  so direct substitution can be used to determine the limit.

↳ Limits with Radicals

### Example 7

Evaluate  $\lim_{x \rightarrow 4} \frac{x-4}{\sqrt{x}-2}$  → Radical in denominator

### Example 8

Evaluate  $\lim_{x \rightarrow 0} \frac{\sqrt{9+x}-3}{x}$  → Radical in numerator

Example 9

Evaluate  $\lim_{x \rightarrow -3} \frac{x+3}{\sqrt{x^2+7}-4}$

Example 10

Evaluate  $\lim_{x \rightarrow 4} \frac{3-\sqrt{5+x}}{1-\sqrt{5-x}}$

## Indeterminate Forms

↳ Limits by Simplifying Rational Expressions

### Example 11

Evaluate  $\lim_{x \rightarrow 0} \frac{\frac{1}{x+4} - \frac{1}{4}}{x}$

### Example 12

Evaluate  $\lim_{x \rightarrow 0} \frac{\frac{1}{(3+x)^2} - \frac{1}{9}}{x}$

Example 13

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Evaluate  $\lim_{x \rightarrow 4} \frac{\frac{1}{\sqrt{x}} - \frac{1}{2}}{x - 4}$

Example 14

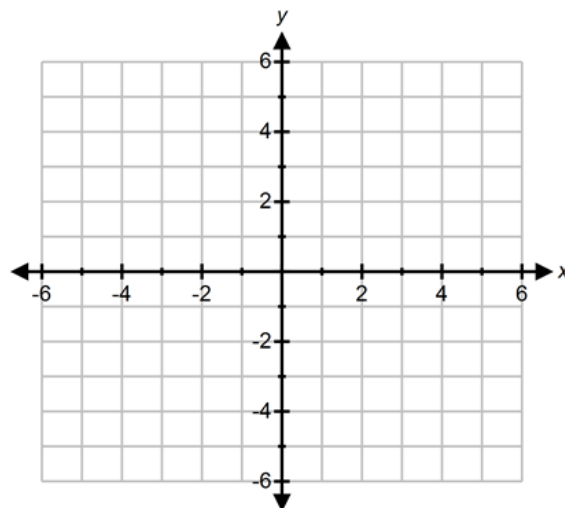
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Evaluate  $\lim_{x \rightarrow 0} \sqrt{x}$

## Absolute Value Function

Review:

$$y = |x|$$



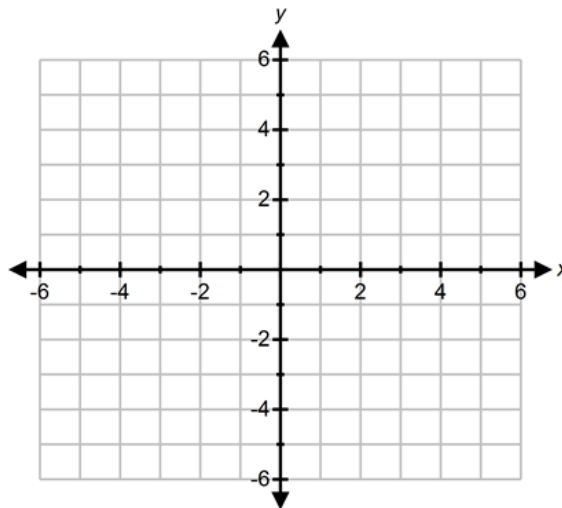
### Example 15

Evaluate  $\lim_{x \rightarrow 0} |x|$



Example 16

Show that  $\lim_{x \rightarrow 0} \frac{|x|}{x}$  does not exist.



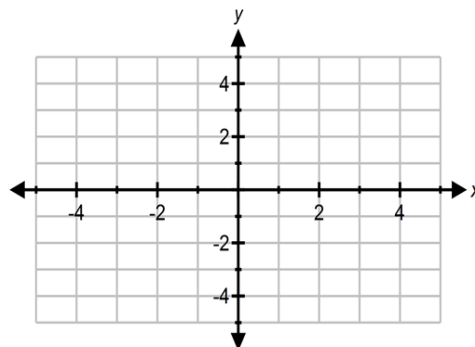
Example 17

Evaluate  $\lim_{x \rightarrow 0} \frac{3x}{2x - |x|}$

## Lesson 2.6 Evaluate limits Indeterminate Form

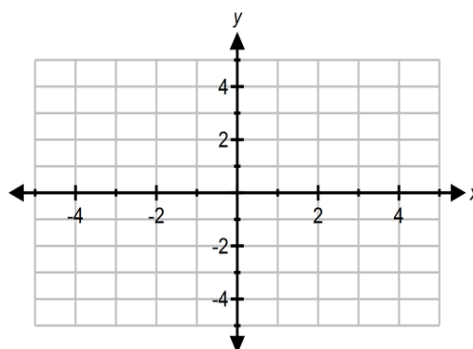
### Example 18

Evaluate  $\lim_{x \rightarrow -2} \frac{x^2 + 3x + 2}{|x + 2|}$



### Example 19

Why is  $|3 - x| = |x - 3|$  ?



### Example 20

Evaluate  $\lim_{x \rightarrow 3^+} \frac{|3 - x|}{2x^2 - 5x - 3}$

Example 21

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Evaluate  $\lim_{x \rightarrow 5} \frac{|5-x|}{2x^2-9x-5}$