Lesson 2.8: Limits at Infinity

(i) limits at infinity of rational functions

(ii) limits at infinity of radical functions

1. Determine the limit of a function as x approaches infinity.
2. Investigate the end behavior of the function using limits to identify possible horizontal asymptotes.

If $\lim_{x\to\infty} f(x) = b$ or $\lim_{x\to-\infty} f(x) = b$

then $y = b$ is a horizontal asymptote

Limits of Rational Functions as $x \to \pm\infty$

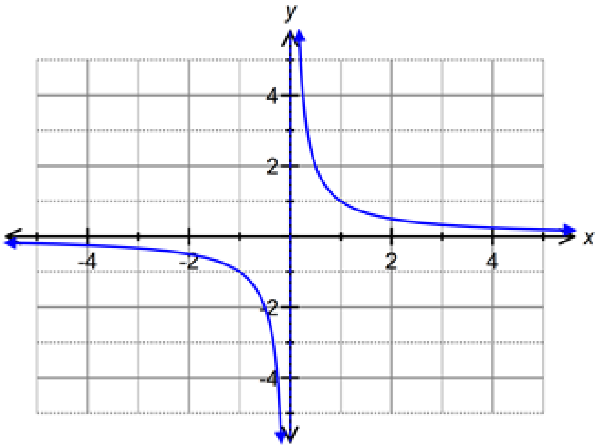
Example 1

Determine the horizontal asympote of $f(x) = \dfrac{1}{x}$

Review:

$$\lim_{x \to 0^+} \frac{1}{x} =$$

$$\lim_{x \to 0^-} \frac{1}{x} =$$



Think About:

$$\lim_{x \to \infty} \frac{1}{x} =$$

x=	1	3	10	100
f(x)=				

$$\lim_{x \to -\infty} \frac{1}{x} =$$

x=	-100	-10	-3	-1
f(x)=				

Horizontal Asymptote:

In order to evaluate limits at infinity, we will need to apply the value of the following limit:

$$\lim_{x \to \pm\infty} \frac{1}{x^n}$$ (Reciprocal Power Function)

$$\lim_{x \to \pm\infty} \frac{1}{x} =$$

$$\lim_{x \to \pm\infty} \frac{1}{x^2} =$$

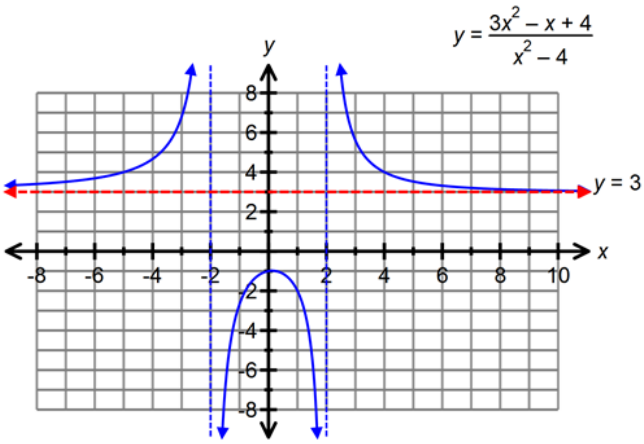
Conclusion: $\lim_{x \to \pm\infty} \frac{1}{x^n}$

Example 2

Determine the $\lim_{x\to\pm\infty} \dfrac{3x^2 - x + 4}{x^2 - 4}$.

Identify the equation of the horizontal asymptote for the graph of the function if it exists.

$$\lim_{x\to\pm\infty} \frac{3x^2 - x + 4}{x^2 - 4}$$



Strategy

Horizontal Asymptote:

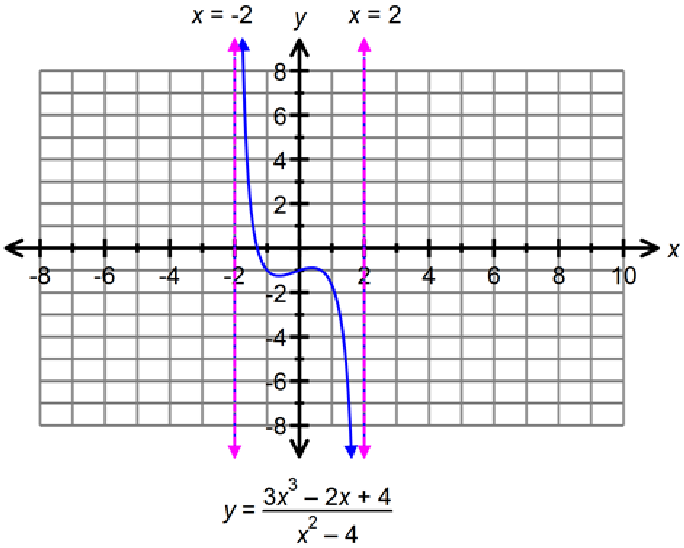
Shortcut:

Example 3

Determine the $\lim_{x \to \pm\infty} \frac{3x^3 - 2x + 4}{x^2 - 4}$.

Identify the equation of the horizontal asymptote for the graph of the function if it exists.

$$\lim_{x \to \pm\infty} \frac{3x^3 - 2x + 4}{x^2 - 4}$$



Horizontal Asymptote:

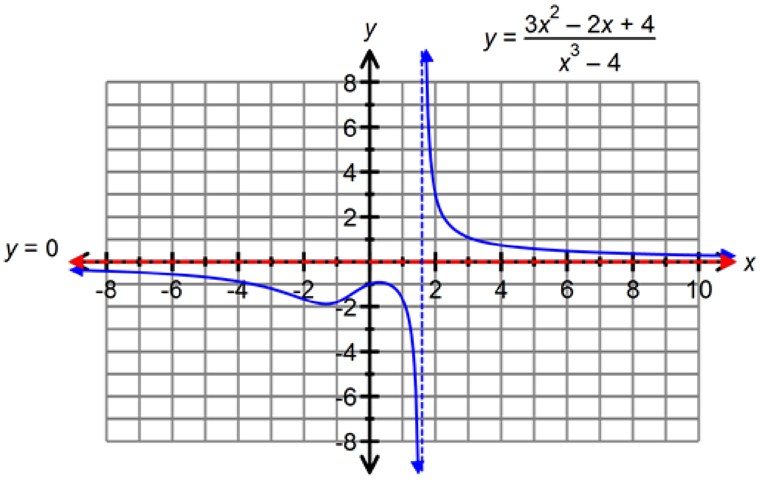
Shortcut:

Example 4

Determine the $\lim_{x \to \pm\infty} \dfrac{3x^2 - 2x + 4}{x^3 - 4}$.

Identify the equation of the horizontal asymptote for the graph of the function if it exists.

$$\lim_{x \to \pm\infty} \frac{3x^2 - 2x + 4}{x^3 - 4}$$



Horizontal Asymptote:

Shortcut:

Summary:

If the degree of the numerator is equal to the degree of the denominator, the horizontal asymptote is the quotient of the leading coefficients.

Example: $\lim_{x \to \pm\infty} \frac{x^2 - 4x}{x^2 - 3x - 4}$

If the degree of the numerator is less than the degree of the denominator, the horizontal asymptote is y = 0.

Example: $\lim_{x \to \pm\infty} \frac{x^3 - 6x}{x^4 + 1}$

If the degree of the numerator is greater than the degree of the denominator, there is no horizontal asymptote.

Example: $\lim_{x \to \pm\infty} \frac{x^2 - 2x - 3}{x + 3}$

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Limits of Radical Functions as $x \to \pm\infty$

★ Example 5

(i) Determine the horizontal asymptotes of the function $y = \dfrac{\sqrt{x^2+1}}{2x-3}$

Note: $\sqrt{x^2} = |x| = \begin{cases} x \text{ if } x \geq 0 \\ -x \text{ if } x < 0 \end{cases}$

(ii) Determine the vertical asymptotes of the function $y = \dfrac{\sqrt{x^2+1}}{2x-3}$

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Your Turn

Determine the horizontal and vertical asymptotes for the function

$$y = \frac{\sqrt{x^2 + 5} - 3}{-x + 1}$$

Your Turn

Evaluate $\lim\limits_{x \to -\infty} \dfrac{\sqrt{5x^6 - x^3 + 1}}{7x^3 - 2x + 9}$