Lesson 3.2: Graph a Rational Function

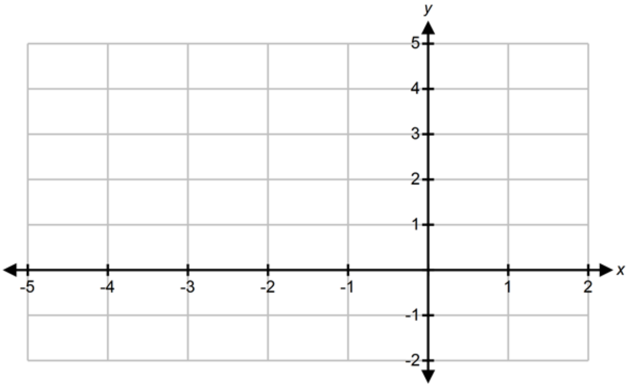
- Behavior Near Asymptotes
- Sign Diagrams
- Complete Analysis and Curve Sketching

Example 1

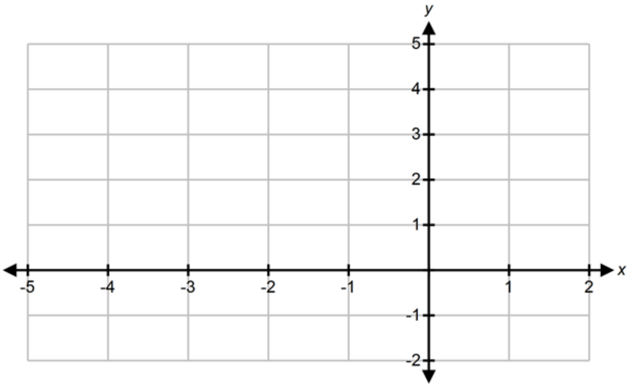
$$f(x)=\frac{x^2-2x-3}{x^2-9}$$ → Function Behavior Near Vertical Asymptotes

Vertical Asymptote

Approaching the Asymptote from the left



Approaching the Asymptote from the right



Sign Diagrams → Determine the x-values where the function (y-value) might change sign.

$$f(x)=\frac{x^2-2x-3}{x^2-9}=\frac{(x+1)(x-3)}{(x+3)(x-3)}$$

Points of discontinuities	Vertical Asymptotes	x-intercepts

Sign Diagram

Sketch a Graph for the Function

→ make use of the sign diagram

$$f(x)=\frac{x^2-2x-3}{x^2-9}$$

We already have:

Point of Discontinuity

Vertical Asymptote

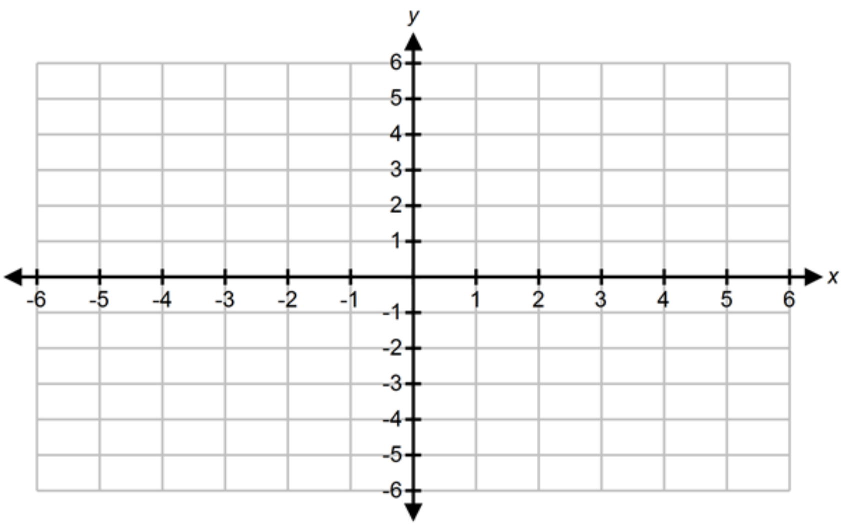
Behavior Near VA

x-intercept

Horizontal Asymptote

↓ describes the end behavior of the function; check to see if the function crosses the HA

y-intercept:



Example 2

Graph and analyze $f(x)=\dfrac{x^2-4x}{x^2-3x-4}$

1. Simplify the expression where possible

2. Determine the x and y-intercepts

3. Determine the non-permissible values:

   a) point(s) of discontinuity

   b) vertical asymptote(s)

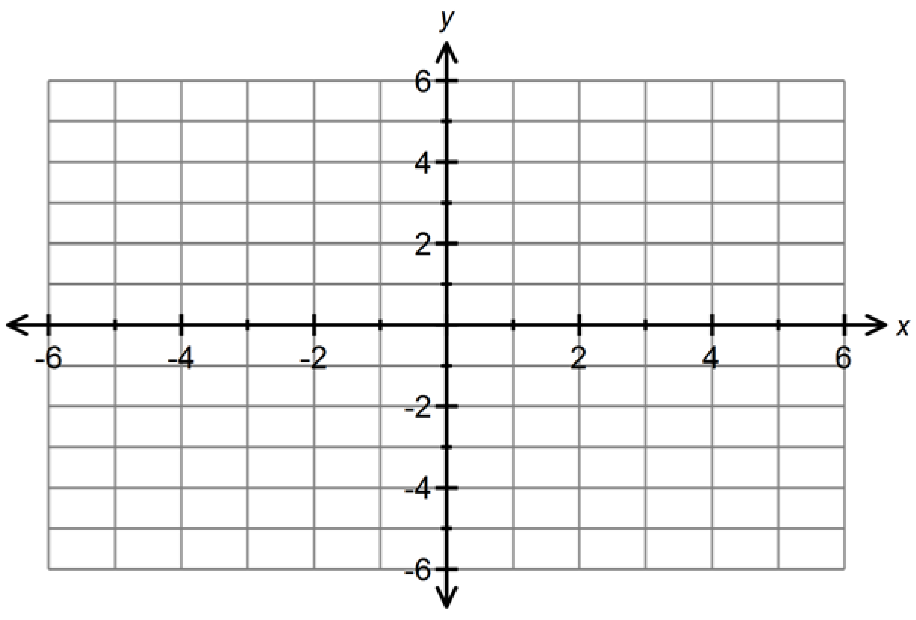
4. Behavior near the vertical asymptote

5. Determine the Horizontal/Oblique Asymptote

$$f(x) = \frac{x^2 - 4x}{x^2 - 3x - 4}$$

6. Draw a Sign Diagram

Use test points to determine where the function (or y-value) is positive or negative for different intervals of x.



Notes:

- When a graph has a hole in it (or a point of discontinuity), the numerator and denominator of the original function contains a common factor

- A factor of only the denominator results in a vertical asymptote

- A factor of only the numerator results in an x-intercept

- Test points on the sign diagram tells us where the function is positive or negative for different intervals of x.

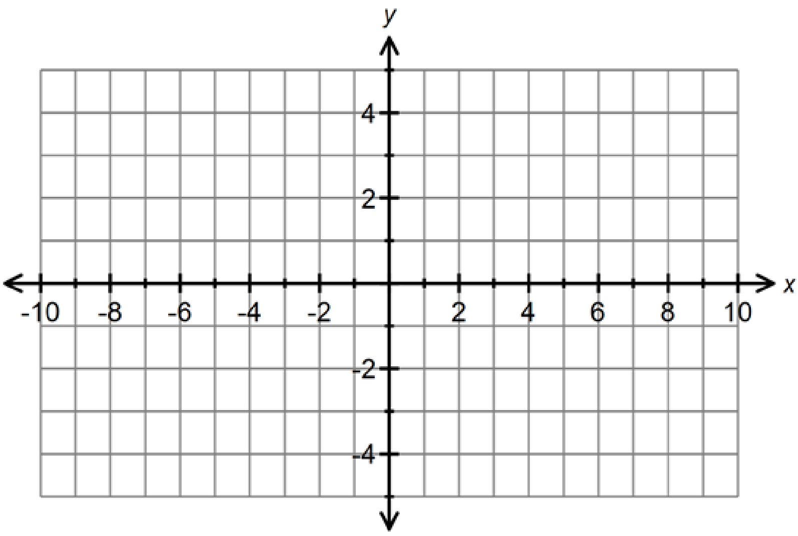
- Horizontal asymptotes describe the end behavior of the function

- Vertical asymptotes describe the behavior of the function near a point.

Example 3

Graph the rational function using intercepts, points of discontinuity, asymptotes, limits, sign diagram and test points.

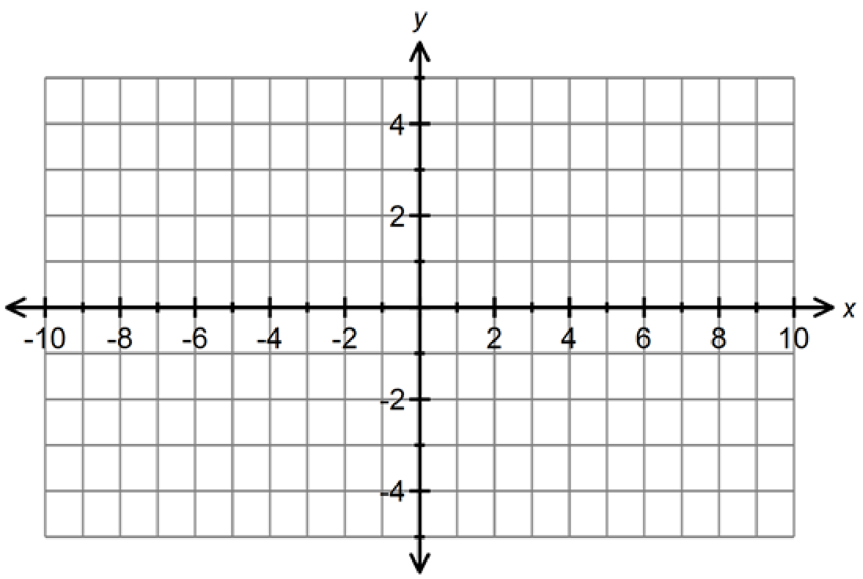
$$y = \frac{x^2 + x - 2}{x^2 + x - 20}$$



Your Turn 1

Graph the rational function using intercepts, points of discontinuity, asymptotes, limits, sign diagram and test points.

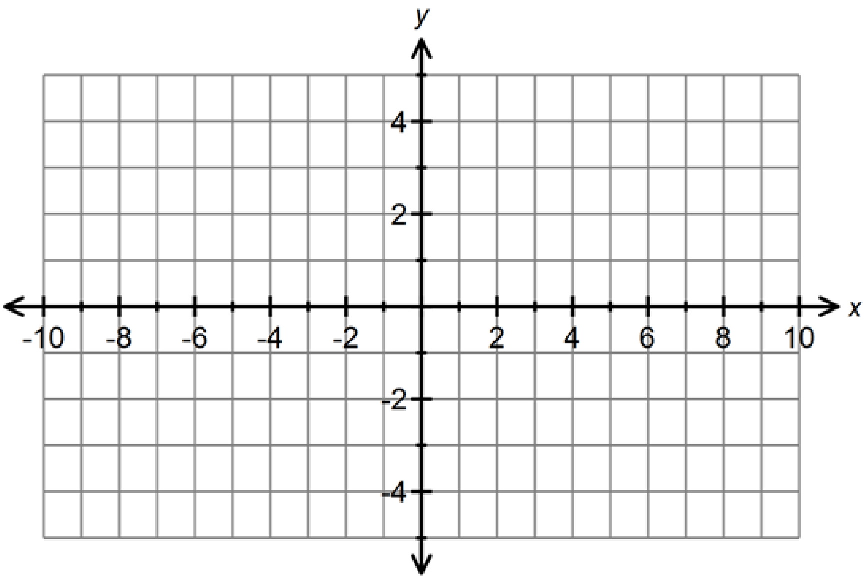
$$y = \frac{9}{x^2 - 9}$$



Your Turn 2

Graph the rational function using intercepts, points of discontinuity, asymptotes, limits, sign diagram and test points.

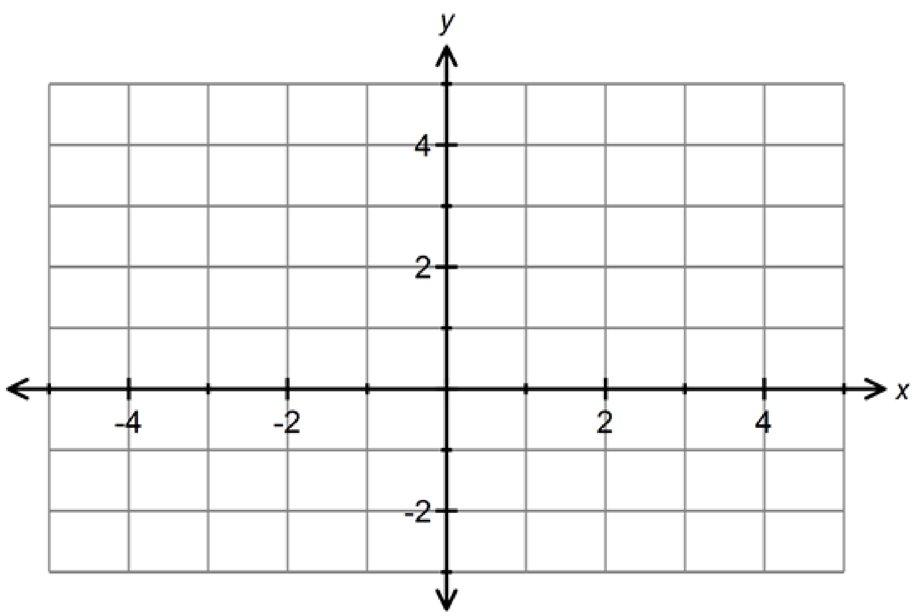
$$y = \frac{2x^2 - x - 6}{x^2 - 4}$$



Your Turn 3

Graph the rational function using intercepts, points of discontinuity, asymptotes, limits, sign diagram and test points.

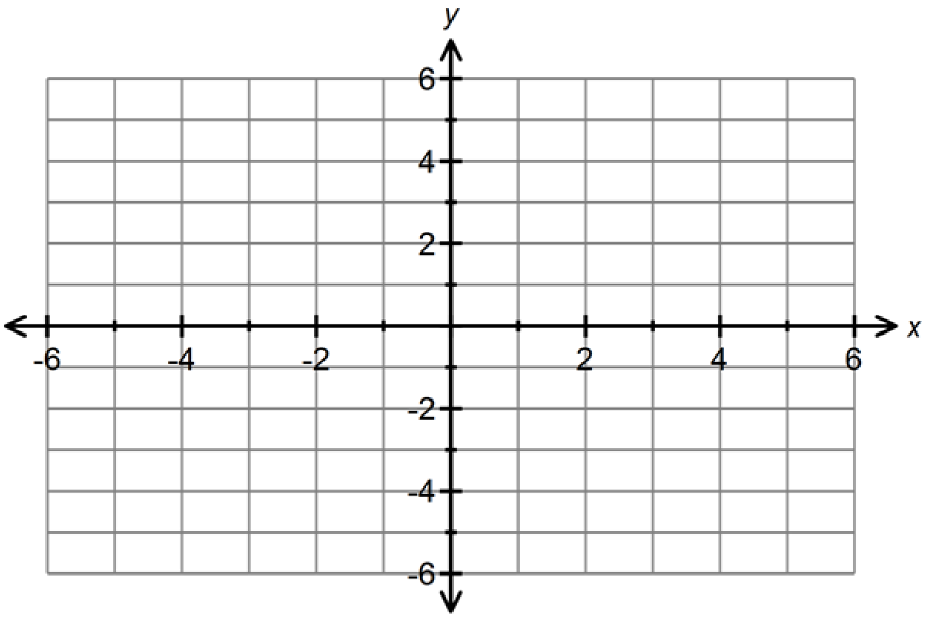
$$y = \frac{8x+3}{4x^2+1}$$



Your Turn 4

Graph the rational function using intercepts, points of discontinuity, asymptotes, limits, sign diagram and test points.

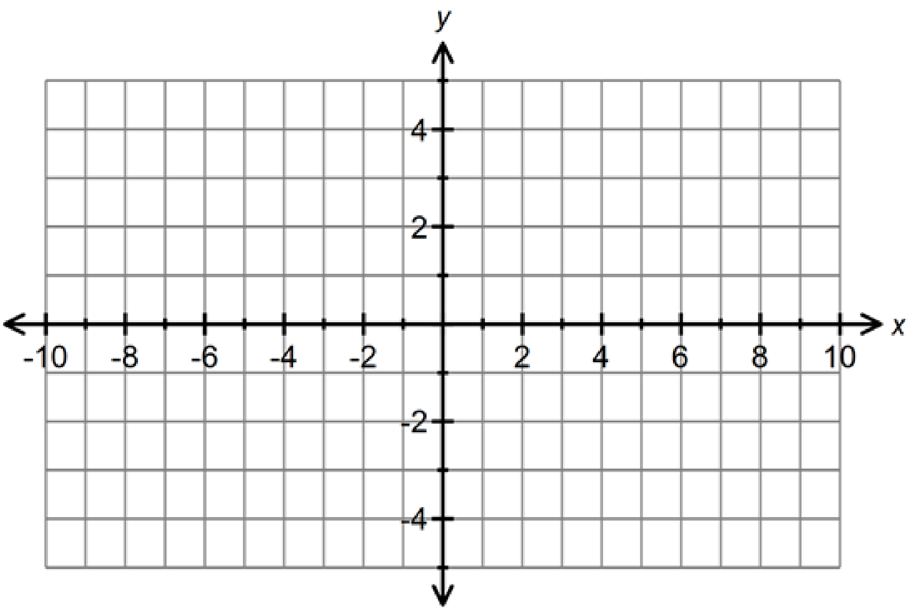
$$y = \frac{x^2 + 2}{x^2 - x - 2}$$



Your Turn 5

Graph the rational function using intercepts, points of discontinuity, asymptotes, limits, sign diagram and test points.

$$y = \frac{x-1}{x^2 - x - 6}$$



Notes:

- A rational function can have many vertical asymptotes.
- If a rational function has a horizontal asymptote, then it does not have an oblique one.
- The graph of a rational function can cross a horizontal asymptote, but does not cross a vertical asymptote.
- Horizontal asymptotes describe the end behavior of the function; vertical asymptotes describe the behavior of the function near a point.

Key Ideas

- We can use all of the information we gather in an analysis of a rational function to sketch its graph.
- It is helpful to determine the behavior of the curve on either side of a vertical asymptote using one-sided limits, to determine if the function is increasing toward ∞ or decreasing toward $-\infty$.
- A sign diagram provides information about regions of the graph that are above or below the *x-axis.*
- It is POSSIBLE for the curve to "cross through" a horizontal asymptote at relatively small values of *x*.
- It is IMPOSSIBLE for the curve to "cross through" a vertical asymptote, as these are found only at non-permissible values of *x*.

Oblique Asymptotes

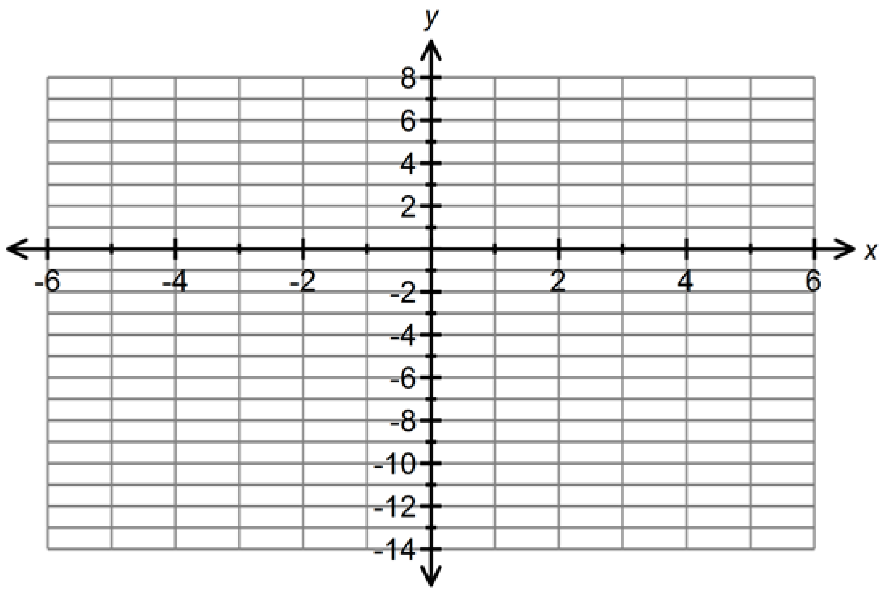
They exist when the degree of the numerator is exactly one more than the degree of the denominator.

oblique asymptote: the quotient from long division

Example 4

Graph the rational function using intercepts, points of discontinuity, asymptotes, limits, sign diagram and test points.

$$y=\frac{-3x^2+2}{x-1}$$



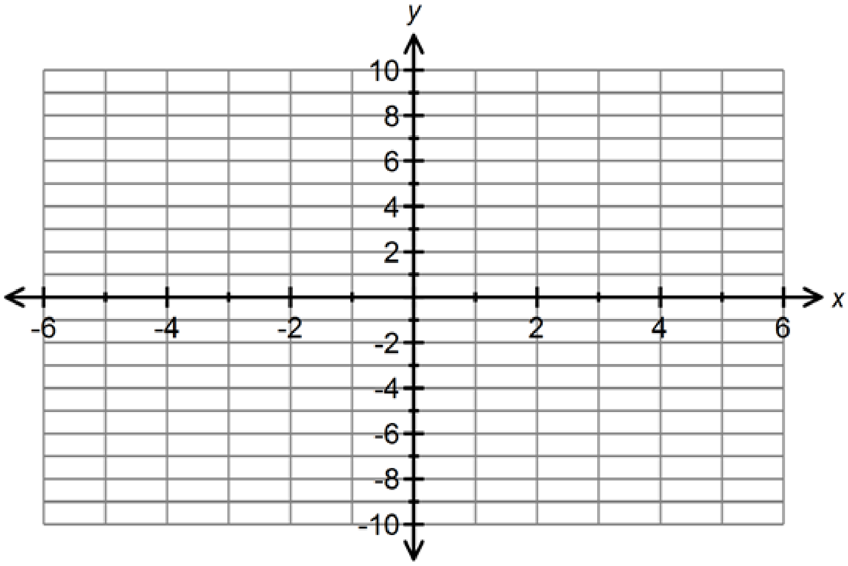
Note:

- If a rational function has a horizontal asymptote, then it does not have an oblique asymptote.

Example 5

Graph the rational function using intercepts, points of discontinuity, asymptotes, limits, sign diagram and test points.

$$y=\frac{x^3+x^2-4x-4}{x^2+x-6}$$



Your Turn 6

Graph the rational function using intercepts, points of discontinuity, asymptotes, limits, sign diagram and test points.

$$y = \frac{x^2 + x - 12}{x - 2}$$