CHAPTER 4

Derivative

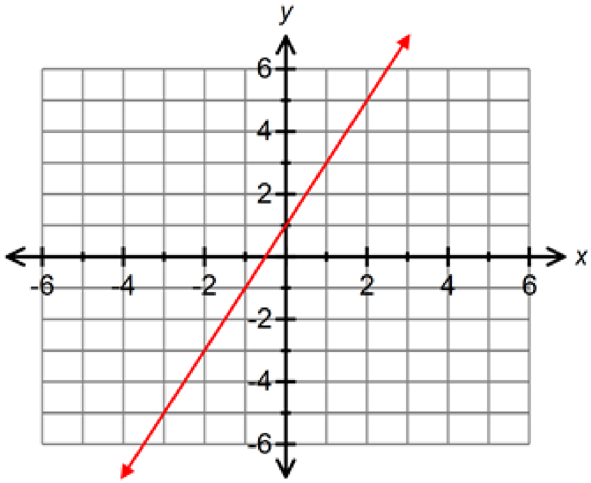
Lesson 4.1: Rates of Change
- average rate (interval of time)
- instantaneous rate (specific time)

Linear Function

↳ Any two points can be used when determining the slope of a line

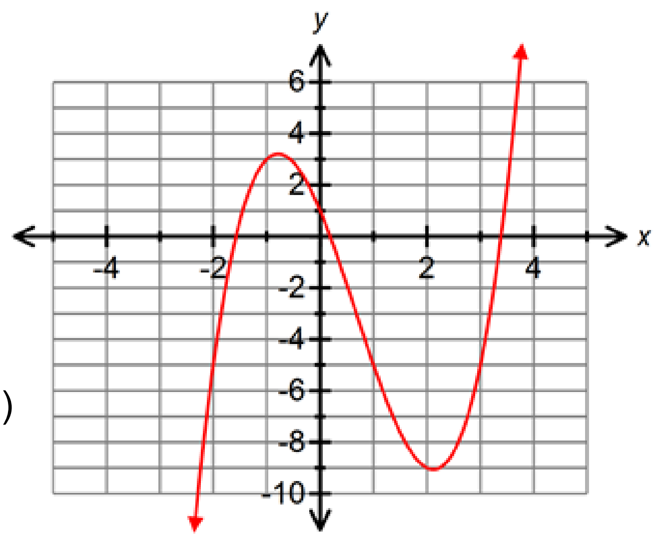
↓

the slope is constant



Non-Linear Function

↳There is no single slope value for this graph.



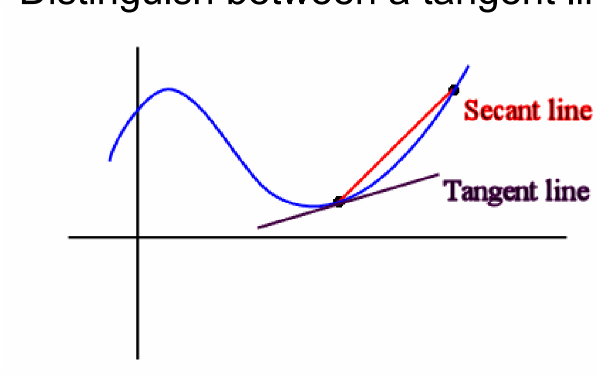
For various intervals of x the

(i) function increases (slope is positive)

(II) levels off (slope is zero) and

(iii) decreases (slope is negative)

Distinguish between a tangent line and a secant line



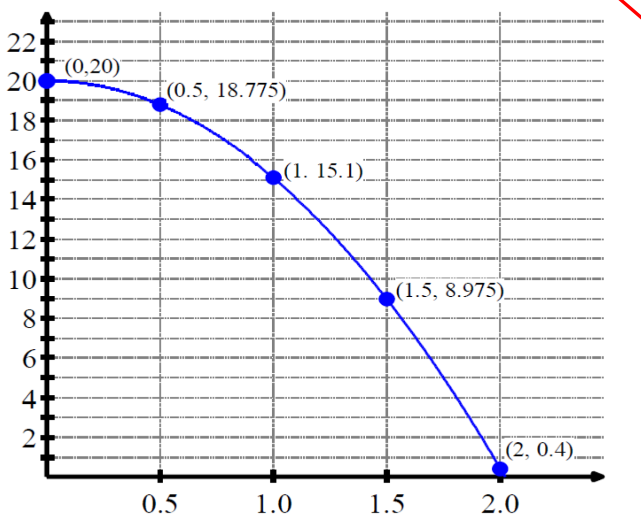
Calculate the slope of a secant line: given two points

Example 1:

An apple falls from atop a 20m tree and is in free-fall as it drops toward the ground.It's height above the ground is a decreasing function of time.

At what average speed is the apple falling between 0.5 sec and 1.5 sec after it starts to drop?

Height (m) vs Time (sec)



$$v_{avg} = \frac{h_2 - h_1}{t_2 - t_1}$$

slope of a secant line → line joining two points on a curve → average rate of change between two points

Average Rate: $m = \frac{f(b) - f(a)}{b - a} = \frac{\Delta y}{\Delta x}$

Calculate the slope of a tangent line: given one point

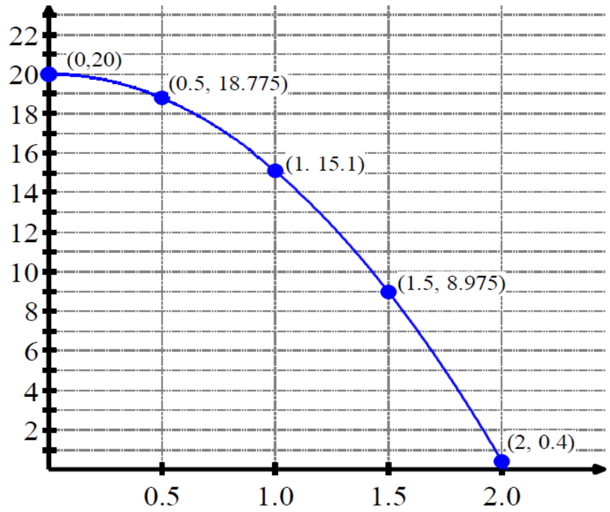
→ need two points to calculate the slope

Example 2:

How fast is the apple falling, exactly 1 sec after it starts to drop?

$$v_{inst} \cong \frac{h_2 - h_1}{t_2 - t_1}$$

points chosen are close to the point of tangency



slope of a tangent line → limiting value of the slope of the secant line → instantaneous rate of change at a point

Instantaneous Rate of Change

Example 3: *Estimate the slope of the tangent line at point P (1,1)*

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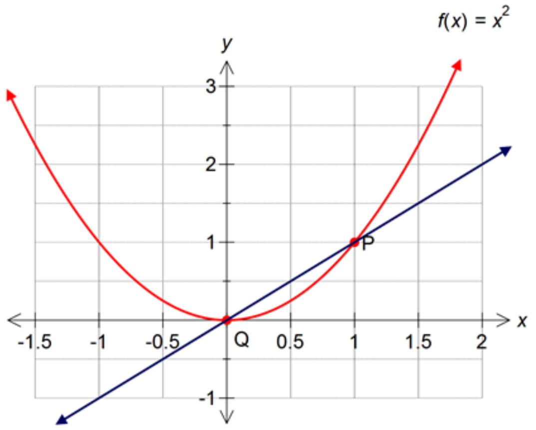
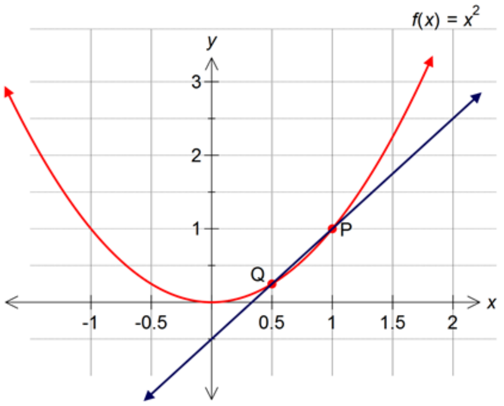
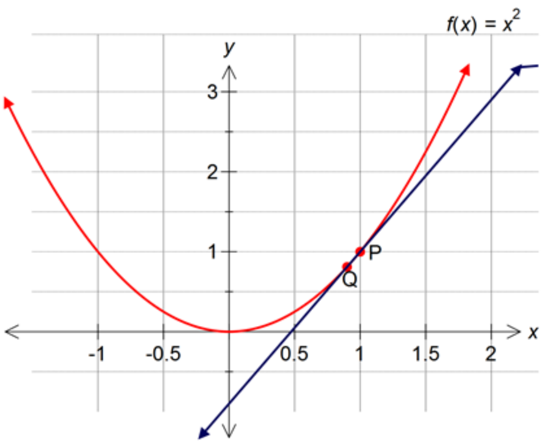
Idea: We need two points to calculate the slope of a tangent line therefore we will estimate the slope of the secant lines close to the point (1,1).



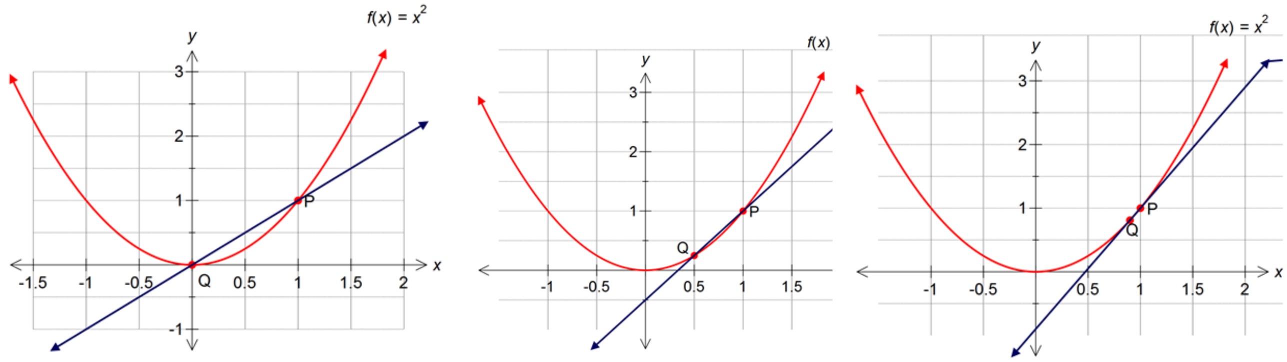
$m_{PQ} =$



$m_{PQ} =$



$m_{PQ} =$



Q approaching P from the left		
x	y	$m_{avg} = \frac{f(b)-f(a)}{b-a}$
0	0	1
0.5	0.25	1.5
0.9	0.81	1.9
0.99	0.9801	1.99
0.999	0.998001	1.999

Q approaching P from the right		
x	y	$m_{avg} = \frac{f(b)-f(a)}{b-a}$
2	4	3
1.5	2.25	2.5
1.1	1.21	2.1
1.01	1.0201	2.01
1.001	1.002001	2.001

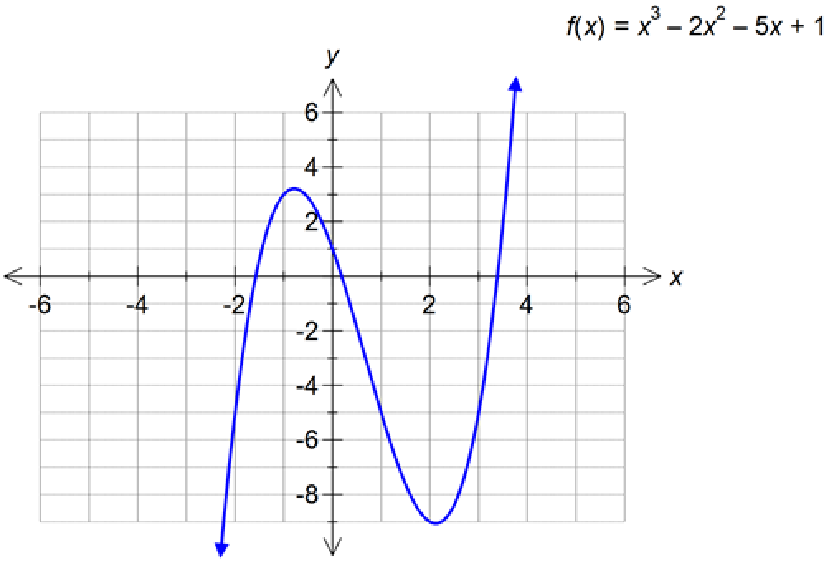
As Q approaches P from either side, the slope of the secant line is approaching a particular value. This value is the slope of the tangent line.

The slope of the tangent line is the limiting value to the slope of the secant lines as point Q moves closer to point P.

$$\lim_{Q \to P} m_{PQ} = m_{\tan gent\ at\ P}$$

Example 4:

Estimate the instantaneous rate of change at x = -1.



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