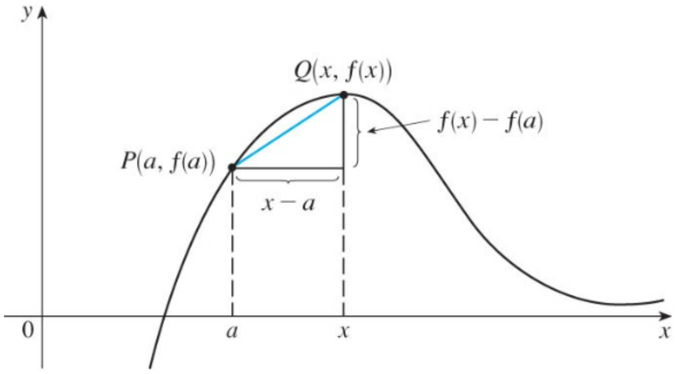
Lesson 4.2: Derivative at a Point

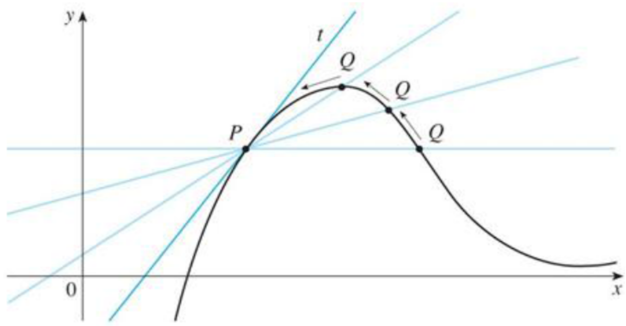
1. Determine the exact value for the slope of a tangent line at a specific point
2. Introduce the slope of a tangent line as the derivative

Exact Slope of a Tangent

Determine the slope of the tangent to the curve $f(x)$ at the point $x = a$



Choose Q as a second point



Move Q to improve the estimate

First Definition:

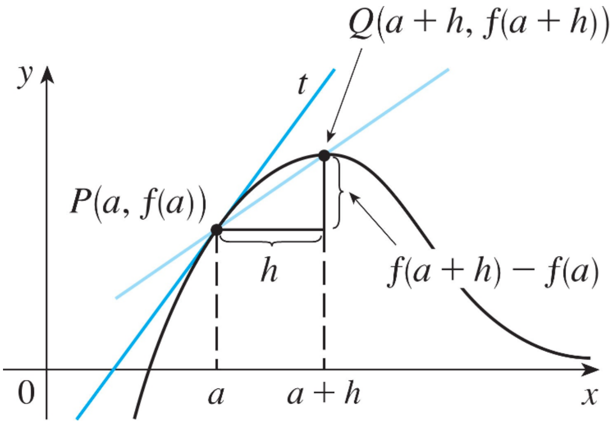
Slope of the Secant: $m_{\sec} = \dfrac{y_Q - y_P}{x_Q - x_P} = \dfrac{f(x) - f(a)}{x - a}$

With Improving Estimates:

$$Q \to P$$
$$x \to a$$
$$f(x) \to f(a)$$

Slope of the Tangent (The derivative): $m_{\tan} = \lim\limits_{x \to a} \dfrac{f(x) - f(a)}{x - a}$

Second Definition:



The variable h represents the distance between the x-coordinates a and $a+h$

$$m_{\sec} = \frac{f(a+h)-f(a)}{(a+h)-a} = \frac{f(a+h)-f(a)}{h}$$ ⟶ slope of the secant line

What happens as the distance between the two points on the curve decreases?

$$m_{\tan} = \lim_{h\to 0}\frac{f(a+h)-f(a)}{h} = f'(a)$$ ⟶ slope of the tangent line

↓

definition of the derivative at a point $x = a$

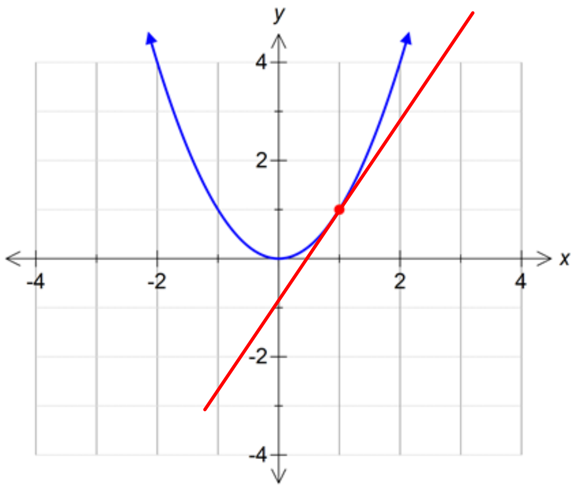
Other Notation:

$$m_{\tan} = \lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x} = \frac{dy}{dx}$$ ⟶

Example 1:

Determine the derivative of the function $f(x) = x^2$ at the point where x = 1.

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$



Alternative Method: (use later when proving differentiability)

$$\lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

→

Example 2

Use the definition of the derivative to find $\left.\frac{dy}{dx}\right|_{x=2}$ for the function $y = 7x - 4x^2$

Example 3: Determine the slope of the tangent line to the curve $f(x) = \sqrt{2x+4}$ where $x = 6$.

→

Example 4:

Determine the derivative of the function $f(x)=\dfrac{2x+1}{3x-1}$ at the point where x = 1.

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Review of Linear Functions and Their Equations

Slope → $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

Slope Intercept Form: $y = mx + b$

Point Slope Form: $y - y_1 = m(x - x_1)$

Parallel Lines → slopes of the lines are equal

Perpendicular Lines → slopes of the lines are negative reciprocals of each other

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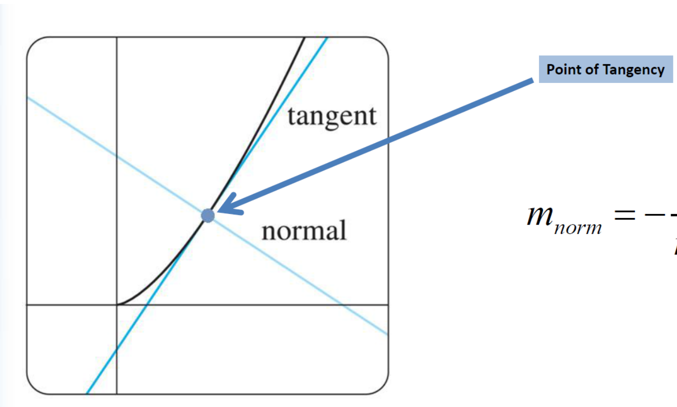
Tangent Line and Normal Line



$$m_{norm} = -\frac{1}{m_{\tan}}$$

Equation of Tangent Line

In Slope-Point Form:

$$y = m(x - x_1) + y_1$$

$y_1 \rightarrow f(a)$

$x_1 \rightarrow a$

$m \rightarrow f'(a)$

Equation of Tangent Line:

$$y = f'(a)(x - a) + f(a)$$

Equation of Normal Line

In Slope-Point Form:

$$y = m(x - x_1) + y_1$$

$y_1 \rightarrow f(a)$

$x_1 \rightarrow a$

$m \rightarrow -\dfrac{1}{f'(a)}$

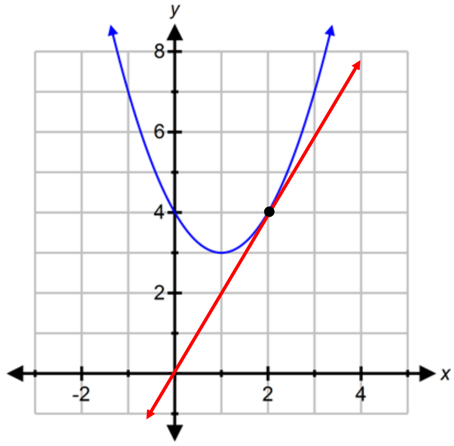
Equation of Tangent Line:

$$y = -\frac{1}{f'(a)}(x - a) + f(a)$$

→

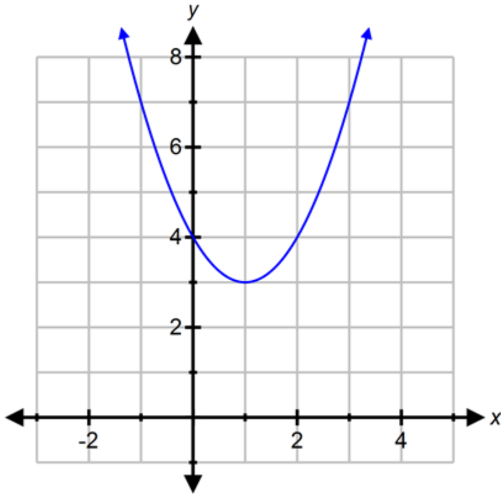
Example 5:

Determine the equation of the tangent line to the curve $y = x^2 - 2x + 4$ at the point (2,4)



Example 6

Determine the equation of the normal line to the curve $y = x^2 - 2x + 4$ at the point (2,4)



Your Turn

(i) Determine the equation of the tangent line to the parabola $y = x^2 - 8x + 9$ at the point (3,-6).

(ii) Determine the equation of the normal line to the curve $y = x - x^3$ at the point (1,0).

(iii) Determine the equation of the tangent line to the graph of at $x = 5$ if $g(5) = -3$ and $g'(5) = 4$

→

(iv)

(a) Determine the equation of the tangent line to the graph of $y = \frac{1}{\sqrt{x}}$ at the point where $x = a$

(b) Find the equations of the tangent lines at the points (1,1) and $\left(4, \frac{1}{2}\right)$

Questions:

p.80-83 #3, 5, 7ab,17, 21, 25