Lesson 4.3: Derivative as a Function

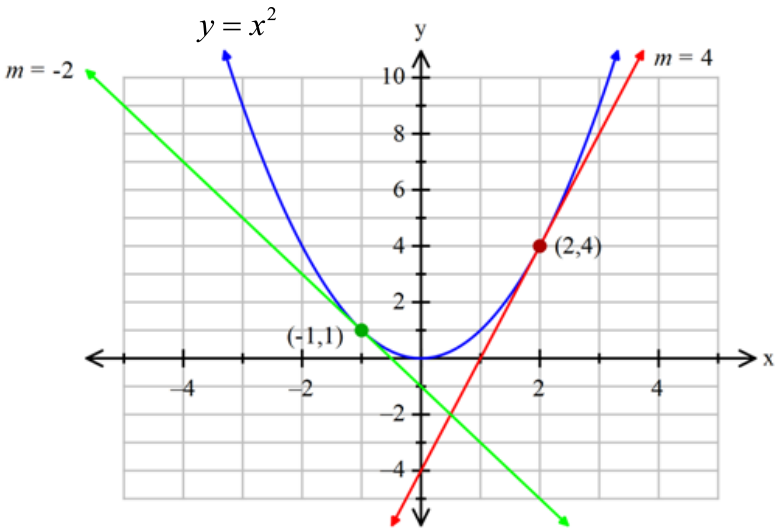
Determine the slope of the tangent line for
any x-value (limited to polynomials of degree 3,
square root, and rational functions with linear terms)

Review:

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

$f'(2) = 4$

$f'(-1) = -2$



The slope of the tangent line is different at different points on the curve

Goal: Given a function, define a derivative function for any x-value.

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

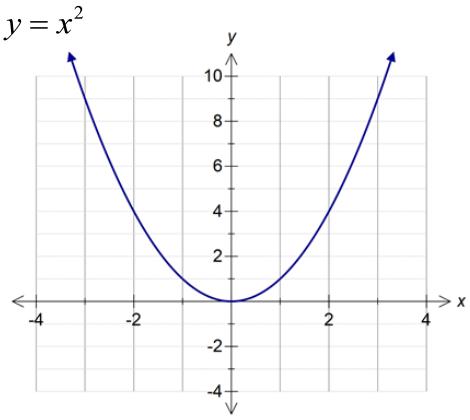
$$\downarrow$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Definition of Derivative
(Method of First Principles)

Example 1:

Find $f'(x)$ if $f(x) = x^2$



$f'(2) = 4$

$f'(0.5) = 1$

$f'(-1) = -2$

→

Example 2

Determine $f'(x)$ for the function $f(x) = x^3 - x^2$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Example 3

Determine $f'(x)$ for the function $f(x) = (x-1)^2$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Example 4

Determine $f'(x)$ for the function $f(x) = \dfrac{x+3}{2x+1}$

Example 5

Determine $f'(x)$ for the function $f(x) = \sqrt{4+3x}$

Worksheet

The Meaning of the Derivative

In Words:

The Derivative Function = A formula for the slope of the tangent at any value of *x* = A formula for instantaneous rate of change = The Slope Function

Notation:

$$f'(x) = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \frac{dy}{dx} = \frac{d}{dx}\left(f(x)\right) = D_x\left(f(x)\right)$$

Derivative Notation

Function	Newton/Lagrange	Leibniz	
$f(x)$	$f'(x)$	$\frac{d}{dx}(f(x))$	$\frac{df}{dx}$
y	y'	$\frac{dy}{dx}$	
x^2+1	$(x^2+1)'$	$\frac{d}{dx}(x^2+1)$	