Lesson 4.4: Differentiability

↳ A function is differentiable if we can compute its derivative

Differentiable at a Point

↳ • a function is differentiable at a point if $f'(a)$ exists

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

• the value $f(a)$ must exist (otherwise the formula does not work)

• there exists only one tangent to the graph of the function at the point $(a, f(a))$

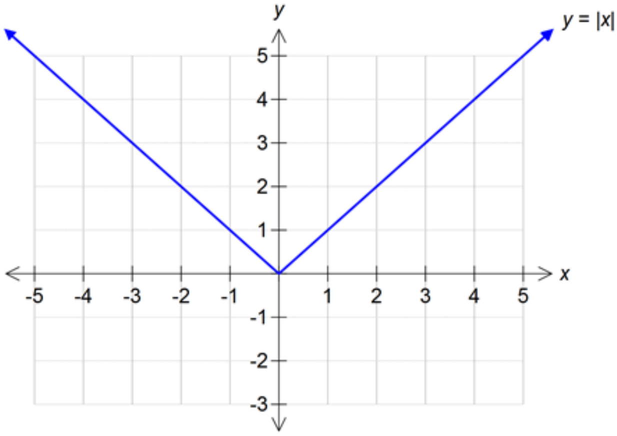
When is a function not differentiable?

↳
- corner point (cusp)
- vertical tangent
- point of discontiunity

→

1. Corner Point

Show that the function $f(x)=|x|$ is continuous at $x=0$ but not differentiable at $x=0$. (i.e., Check $f'(0)$)

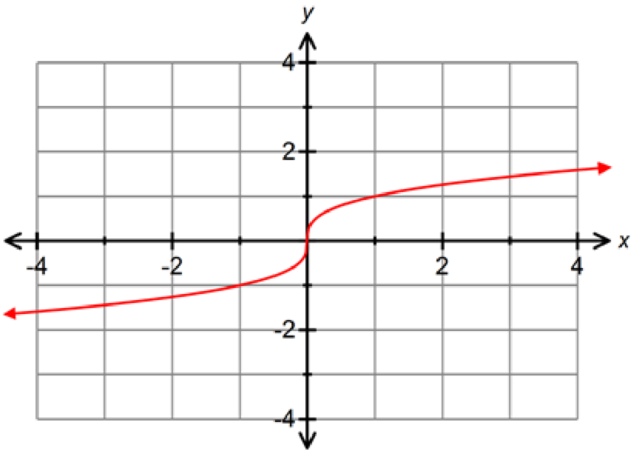


$$|x| = \begin{cases} x \text{ if } x \geq 0 \\ -x \text{ if } x < 0 \end{cases}$$

→

2. Vertical Tangent

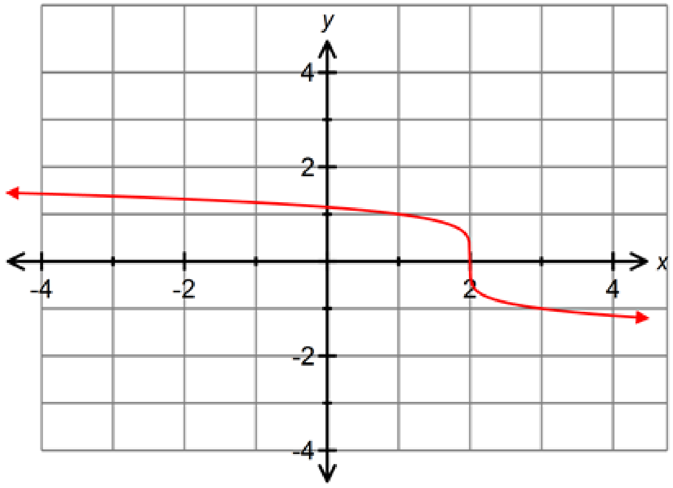
$y = x^{\frac{1}{3}}$



secant line begins to take on the slope of a vertical line

Show the function is not differentiable at x = 0?

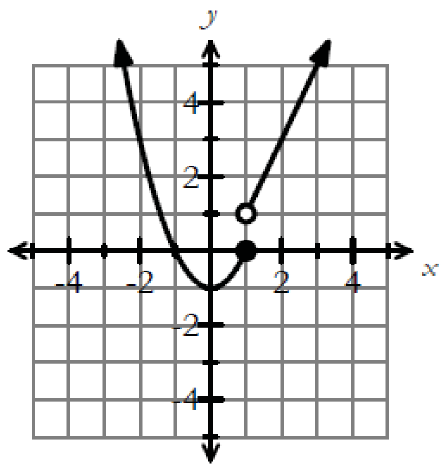
Example: $f(x) = \sqrt[5]{2-x}$



3. Point of discontinuity

Is this function continuous at x = 1?

Is the function differentiable at x = 1?



Notes:

1) If a function is differentiable at a, then it is continuous at a .

2) If a function is continuous at a does not necessarily mean it is differentiable at a .

3) Functions that contain a corner point, cusp, vertical tangent or a discontinuity are not differentiable.

→

Example: $f(x)=\begin{cases} x^2-x-2, & x\le 2 \\ \dfrac{x-2}{x}, & x>2 \end{cases}$

Show that $f(x)$ is not differentiable at x=2.

→

Your Turn

Determine whether $f(x)$ is differentiable at x = -1?

$$f(x) = \begin{cases} 5x - 3x^2, & x \geq -1 \\ 9x + 1, & x < -1 \end{cases}$$