Lesson 4.5: Derivative Rules

Learning Objectives:

1. Apply the derivative rules to determine the derivative of a function.
- constant
- constant multiple
- sum and difference
- product
- quotient

Constant Rule $f(x) = c$

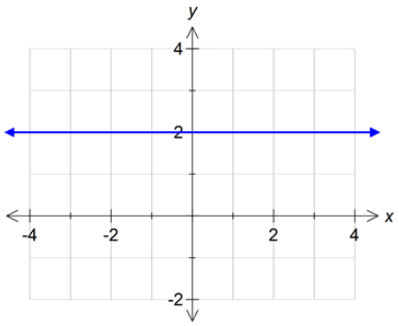
Use the definition of the derivative to prove $f'(x) = 0$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Verify:

Example $f(x) = 2$

$f'(x) =$



Constant Multiple Rule $g(x) = cf(x)$

Use the definition of the derivative to prove $g'(x) = cf'(x)$

$$g'(x) = \lim_{h \to 0} \frac{g(x+h) - g(x)}{h}$$

Example

$$g(x) = 2f(x)$$

$$g'(x) = 2f'(x)$$

Sum and Difference Rule $k(x) = f(x) + g(x)$

$k(x) = f(x) - g(x)$

Use the definition of the derivative to prove $k'(x) = f'(x) - g'(x)$

$$k'(x) = \lim_{h \to 0} \frac{k(x+h) - k(x)}{h}$$

→

Product Rule $k(x) = f(x) \times g(x)$

Use the definition of the derivative to prove $k'(x) = f'(x)g(x) + f(x)g'(x)$

$$k'(x) = \lim_{h \to 0} \frac{k(x+h) - k(x)}{h}$$

→

Quotient Rule $Q(x) = \dfrac{f(x)}{g(x)}$ where $g(x) \neq 0$

Use the product rule to prove $Q'(x) = \dfrac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$

→

Power Rule $\frac{d}{dx}(x^n) = nx^{n-1}$

Distinguish between the functions:

$y = x^2$ and $y = 2^x$

Example: Determine the derivative $f(x) = x^7$ of using the power rule and then verify using the product rule.

Power Rule

Product Rule

Note: The derivative of the product is not the product of the derivative

Examples: Determine the derivative of the following:

(a) $y = 4x^3 - 3x^2 + 5$

(b) $y = \sqrt{x} + x^3$

(c) $y = x^3\sqrt{x}$

(d) $y = \sqrt{x} + \frac{2}{\sqrt{x}} + \frac{3}{\sqrt{\pi + 3}}$

➡

(e) $y = (x-2)(2x+3)$ (two methods)

Think about:

$y = (4x^3 + 2x^2 + 5x)(-x^2 - 3x + 11)$

(f) $y = \dfrac{3x+1}{2x-5}$

(g) $y = \dfrac{6-\sqrt{x}}{1+\sqrt{x}}$

→

(h) Evaluate $(f \times g)'(2)$ given the following information:

$$f(2) = 5,\ f'(2) = 6,\ g(2) = 7,\ g'(2) = -1$$

(i) $y = \dfrac{\sqrt{x} + 3x^2}{x}$

(j) $y = \dfrac{5x^2 - \dfrac{4}{x}}{2}$

Your Turn: Differentiate the following:

(a) $y = -2x^3 + 3x^2$

(b) $y = 80 + 64t - 4.9t^2$

(c) $y = \dfrac{x^4 + 3x^2 + 7}{x^2}$

(d) $y = \dfrac{x^2 - 4}{2x + 3}$

(e) $y = 4x^2 - \frac{x^2}{7} - \sqrt{2}x + \frac{3}{x} + 7\pi^2$

(f) $y = \frac{x^{\frac{3}{4}} - x^3}{x^2}$

(g) $y = (3x^3 - 2x)(x^4 + 2x^3)$

(h) $g(x) = x^2\left(\frac{2}{x} - \frac{1}{x+1}\right)$

(i) $f(x) = \frac{2 - \frac{1}{x}}{x-3}$

(j) $f(x) = \sqrt[3]{x}(\sqrt{x} + 3)$

(k) $y = \dfrac{x^3 + 3x + 2}{x^2 + 1}$

(l) $y = \dfrac{t^6 - 5\sqrt[4]{t} + t^5}{t^5\sqrt{t}}$