# Lesson 4.6: Chain Rule

→ used when differentiating composite functions $f(g(x))$

## *Chain Rule using Function Notation*

$$y = f(g(x)) \longrightarrow y' = f'(g(x)) \cdot g'(x)$$

Example 1: Expand the binomial and differentiate using the power rule on each term.

| $y=(4x-5)^2$ | $y=(x^2+1)^3$ |
| --- | --- |
| | |

Rewrite as a composition of functions

$y=(4x-5)^2$

$f(x) =$

$g(x) =$

Rewrite as a composition of functions

$y=(x^2+1)^3$

$f(x) =$

$g(x) =$

Example 2: Determine the derivative of each function using the chain rule.

$a)\ y=\sqrt{4x^3+5x-2}$

$b)\ y=(3x^2+1)^{10}$

$c)\ f(x)=\dfrac{1}{\sqrt[3]{x^2+x+1}}$

## Example 2: Cont'd

$d)\ y=\left[x^2+(1-3x^4)^5\right]^3$

$e)\ f(x)=\dfrac{3}{5x^4}-\dfrac{1}{\sqrt{2x}}+\dfrac{1}{(3x)^{-1}}$

Example 3: Determine the derivative of each function using the chain rule with a product.

$a)\ y=\dfrac{2x}{(4-5x)^2}$

$b)\ y=(x^2+1)^3(x^2+2)^6$

→

Example 3: Cont'd

$c)\ y=(x^3-3x^2)^7(3x^4-7)^5$

$d)\ y=\sqrt{(5x-4)(3x^3-4x^2+2x-3)}$

Example 4: Determine the derivative of each function using the chain rule with a quotient.

$a)\ y=\dfrac{(x^3-4x^2)^8}{(2x^5-1)^6}$

$b)\ y=\left(\dfrac{x^2+1}{x^2-1}\right)^3$

$c)\ y=\sqrt[3]{\dfrac{x^2-4x+3}{3x+2}}$

→

## *Chain Rule using Leibniz Notation*

$$\left.\begin{aligned} y &= f(u) \\ u &= g(x) \\ y &= f(g(x)) \end{aligned}\right\} \longrightarrow \frac{dy}{dx} = \frac{dy}{du} \bullet \frac{du}{dx}$$

Example: $y = (x^2 + 1)^3$

Write y as a function of u
Write u as a function of x

} used when we look at word problems involving rates of change

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