Lesson 4.7: Higher Order Derivatives

└→ understanding higher order derivatives as rates of change

Notation

$y = f(x)$

First Derivative

$$\frac{dy}{dx} = f'(x)$$

Second Derivative

$$\frac{d^2y}{dx^2} = f''(x)$$

Third Derivative

$$\frac{d^3y}{dx^3} = f'''(x)$$

nth Derivative

$$\frac{d^ny}{dx^n} = f^{(n)}(x)$$

Note:

└→ the parentheses in the exponent denotes differentiation

$$f^{(2)}(x) = f''(x)$$

└→ the absence of parentheses in the exponent denotes an exponent

$$f^2(x) = [f(x)]^2$$

Second Derivates and Rates of Change - An Example

Original Function	First Derivative	Second Derivative
$y = f(x)$	$\frac{dy}{dx} = f'(x)$	$\frac{d^2y}{dx^2} = f''(x)$

Position	Speed	Second Derivative
$s = f(t)$	$v = \frac{ds}{dt} = f'(t)$	$a = \frac{dv}{dt} = \frac{d^2s}{dt^2} = f''(t)$

→

Example 1

Computing a Second Derivative

Find $f''(x)$ for the function $f(x) = 3x^2 - 7x + 5$

Example 2

Find $\dfrac{d^2y}{dx^2}$ for the function $f(x) = (2x+3)^7(3x-7)^5$

Example 3

Find $f''(x)$ for the function $f(x) = \sqrt[3]{3x+7}$

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Your Turn

Find $f''(x)$ for the function $f(x)=\sqrt{\dfrac{2x-1}{x+3}}$

Example 4

Given $f(x) = x^5 + 3x^3 + 42$, determine the value of *n*, such that $f^{(n)}(x) = 0$.

Example 5

Determine the point on the graph of $f(x) = -x^3 + 12x^2 + 4x - 10$ where $f''(x) = 0$

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Simple Applications of Derivatives

Motion

$s'(t) = v(t)$

$s''(t) = v'(t) = a(t)$

object is not moving: $v(t) = 0$

object is moving to the right or forward: $v(t) = \textit{positive value}$

object is moving to the left or backward: $v(t) = \textit{negative value}$

there is no change in the velocity: $a(t) = 0$

object is going faster (velocity is increasing) $a(t) = \textit{positive value}$

object is going slower (velocity is decreasing) $a(t) = \textit{negative value}$

Example 6

If a stone is dropped from a cliff that is 122.5m high, then its height in metres after t seconds is $h = 122.5 - 4.9t^2$.

(a) Find its velocity after 1sec and 2sec.

(b) When will the stone hit the ground?

(c) With what velocity will it hit the ground?

Example 7

The position of a particle moving on a line is given by the equation $s = 2t^3 - 21t^2 + 60t, t \geq 0$ where time is measured in seconds and position in metres.

(a) What is the velocity after 3 sec and after 6 sec?

(b) When is the particle at rest?

(c) When is the particle moving in the positive direction?

(d) Draw a diagram to represent the motion.
Find the total distance travelled by the particle during the first 6 sec.

Example 8

A dynamite blast propels a heavy rock straight up with a launch velocity of 160ft/sec (approximately 109mph). It reaches a height in feet after t seconds given by $s(t) = 160t - 16t^2$.

(a) How high does the rock go?

(b) What is the velocity and speed of the rock when it is 256ft above the ground on the way up? on the way down?

(c) What is the acceleration of the rock at any time during its flight (after the blast)?

(d) When does the rock hit the ground?