Lesson 4.8 Implicit Differentiation

↳ Determine the derivative of a relation using implicit differentiation

Explicit: y is represented in terms of x only

$y + 2x = 3$

$y = \dfrac{\sqrt{3x-17}}{x^2+5x+8}$

Implicit: any equation relating x and y

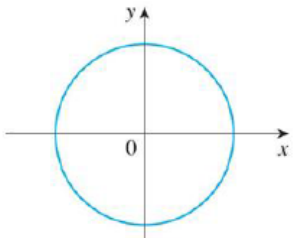
$x^2 + y^2 = (2x^2 + 2y^2 - x^2)^2$

$x^2y^2 - y^3 = xy$

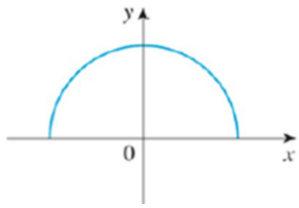
Representing an Implicit Relation Explicitly

Can a circle be an explicit function?

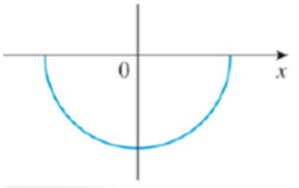
$x^2 + y^2 = 25$ →



$x^2 + y^2 = 25$ → $y = \sqrt{25 - x^2}$ →

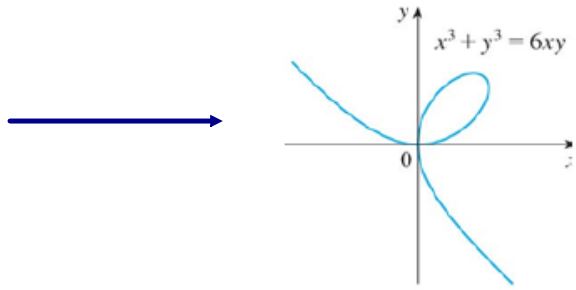


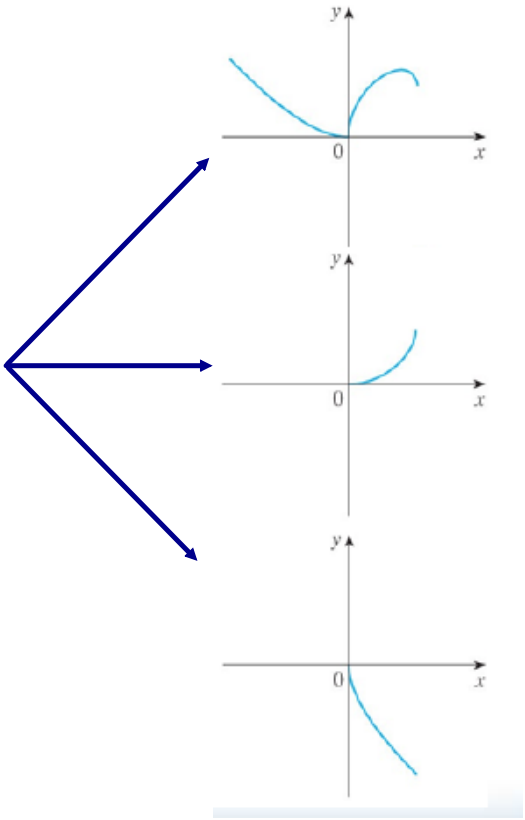
$x^2 + y^2 = 25$ → $y = -\sqrt{25 - x^2}$ →



Some Relations are best Represented Implicity

Descartes' Folium

$x^3 + y^3 = 6xy$ 

$x^3 + [f(x)]^3 = 6xf(x)$ 

How do we Differentiate an Implicit Relation?

Example:

$$x^2 + y^2 = 7$$

Thinking functions: → $y = f(x)$

$$x^2 + f(x)^2 = 7$$

Use the Chain Rule:

$$2x + 2f(x) \cdot f'(x) = 0$$

With Tidier Notation:

$$2x + 2y\frac{dy}{dx} = 0$$

Isolate the Derivative:

$$2y\frac{dy}{dx} = -2x$$

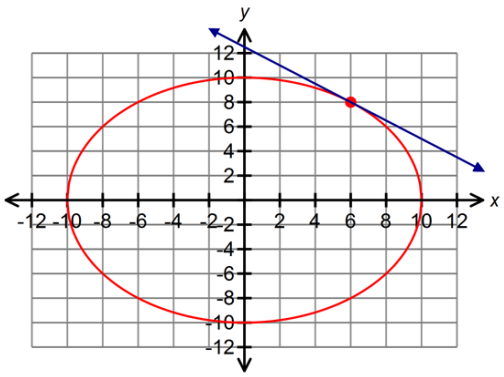
$$\frac{dy}{dx} = -\frac{2x}{2y}$$

$$\frac{dy}{dx} = -\frac{x}{y}$$

→

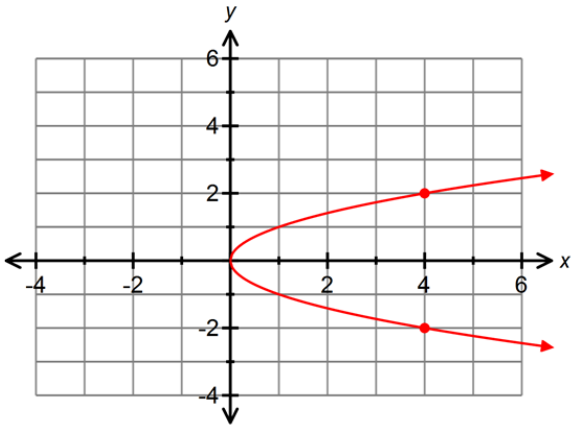
Example 1

Determine the slope of the tangent line $x^2 + y^2 = 100$ to the circle at the point x = 6



Example 2

Find $\frac{dy}{dx}$ if $y^2 = x$



Example 3

Determine $\frac{dy}{dx}$ for $x^2 + xy + y^2 = 5$

Note:

$x^2 + x \cdot f(x) + [f(x)]^2 = 5$

Think about:

1. Differentiate both sides of the equation with repsect ot x
2. Collect the terms with $\frac{dy}{dx}$ on one side of the equation
3. Factor out $\frac{dy}{dx}$
4. Solve for $\frac{dy}{dx}$

Example 4

Determine $\frac{dy}{dx}$ for $x^3 - x^2y = y^2x + y^3$

Example 5

Determine the equation of the tangent line to the curve or $3x^2 - 2xy + xy^3 = 7$ at the point (1,2)

Example 6

Determine $\frac{d^2y}{dx^2}$ if $2x^3 - 3y^2 = 8$