Unit 5: Applications of Derivatives

1) Related Rates

↳ real world examples where things are changing over time (using Implicit Differentiation

2) Curve Sketching (using derivatives)

↳ A) Polynomial Functions

B) Rational Functions

3) Optimization Word Problems

↳ Maximum and Minimum Word Problems

Lesson 5.1: Related Rates

↳ use implicit differentiation to handle questions like:

- Pebble Dropped in Pond
- Edge of Cube Expanding
- Conical Tanks
- Air Being Pumped Into A Ballon
- Ladder Sliding Down a Wall

↳ taking derivatives with respect to time

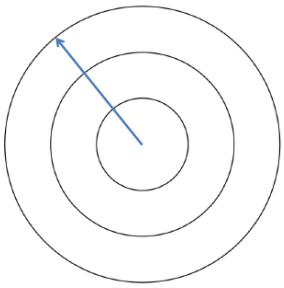
Related Variables

Scenario

↳ A stone is thrown into a pond.

Wave patterns are created in the shape of circles.

Each of the ripples (circles) expands outward from the point of impact.



Radius	Circumference	Area
r	$C = 2\pi r$	$A = \pi r^2$

- Each is a function of time.
- They are all related to each other.

Finding the Relationship Between the Rates

↳ use implicit differentiation to make the connection

$$C = 2\pi r \qquad A = \pi r^2$$

$$\frac{dC}{dt} = 2\pi \frac{dr}{dt} \qquad \frac{dA}{dt} = 2\pi r \frac{dr}{dt}$$

Example 1

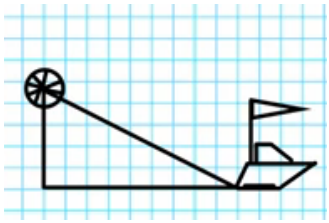
A pebble is dropped into a calm pond.
The radius of the first ripple is 3 cm and is increasing at a constant rate of 1 cm per second.
At what rate is the area of the circle growing?

How to Solve Related Rate Problems [DREDS]

1. Diagram:	Draw a diagram
2. Rates	Which quantities are changing? $\frac{dC}{dt}, \frac{dV}{dt}$.... Is the rate positive or negative?
3. Equation:	Write an equation relating the variables (may use pythagorean theorem, area/volume, proportion or a trig function)
4. Differentiate	Differentiate the equation with respect to time
5. Substitution	Substitute in given values and solve

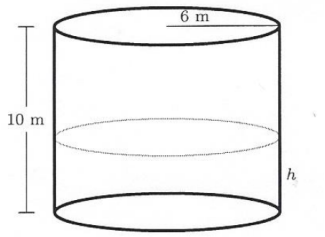
Example 2

A winch 20 feet above sea level is used to reel in a rope connected to a boat at 2 ft/sec. How fast is the boat moving when the rope is 45 feet in length?



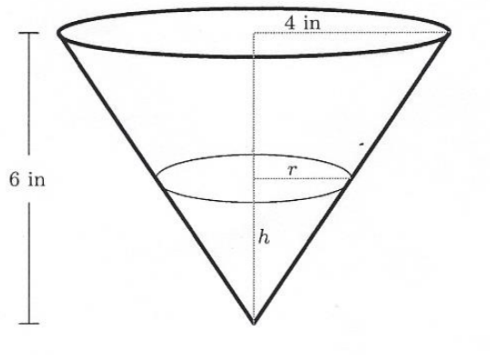
Example 3:

A cylindrical aquarium tank is being filled with water at a rate of $6\frac{m^3}{\text{min}}$. The radius of the tank is 6m and the height is 10m. How fast is the height of the water level increasing?



Example 4:

A cone shaped coffee filter is draining at a rate of $20\frac{in^3}{\text{min}}$.
The filter has a diameter of 8 inches and a height of 6 inches. How fast is the coffee level falling when the coffee is at 3 inches deep?



Example 5:

A 15-foot ladder propped against a wall is sliding along the ground at a rate of $3\frac{ft}{\text{sec}}$. How fast is the ladder sliding down the wall when the base of the ladder is 12 feet from the wall?

Example 6:

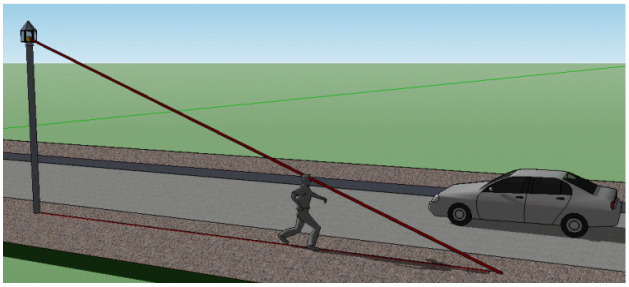
Alice and Bob leave home for school in the morning. Alice walks north at a rate of $8\frac{ft}{\text{sec}}$ while Bob walks east at a rate of $6\frac{ft}{\text{sec}}$. How fast is the distance between Alice and Bob changing when Alice is 20ft and Bob is 15ft from home?

Example 7

A street light is mounted at the top of a 15ft pole. A man 6ft tall walks away from the pole at a rate of $5\frac{ft}{\text{sec}}$.

(A) How fast is the tip of his shadow moving when he is 40ft from the pole?

(B) How fast is the length of his shadow changing when he is 40ft from the pole?



Example 8

A spot light is on the ground 20 feet away from a wall and a 6 feet tall person is walking towards the wall at a rate of 2.5 ft/sec. How fast is the height of the shadow changing when the person is 8 feet from the wall. Is the shadow increasing or decreasing in height at this time?

Useful Formulas:

Area of a triangle: $A = \frac{1}{2}bh$
Area of a rectangle: $A = lw$
Area of a circle: $A = \pi r^2$
Circumference of a circle: $C = 2\pi r$

Volume of a sphere: $V = \frac{4}{3}\pi r^3$
Surface Area of a sphere: $A = 4\pi r^2$

Volume of a cylinder: $V = \pi r^2 h$
Surface Area of a cylinder: $A = 2\pi rh + 2\pi r^2$

Volume of a cone: $V = \frac{1}{3}\pi r^2 h$

Volume of a rectangular prism: $V = lwh$
Surface Area of a rectangular prism: $A = 2lw + 2lh + 2wh$