Lesson 5.3:

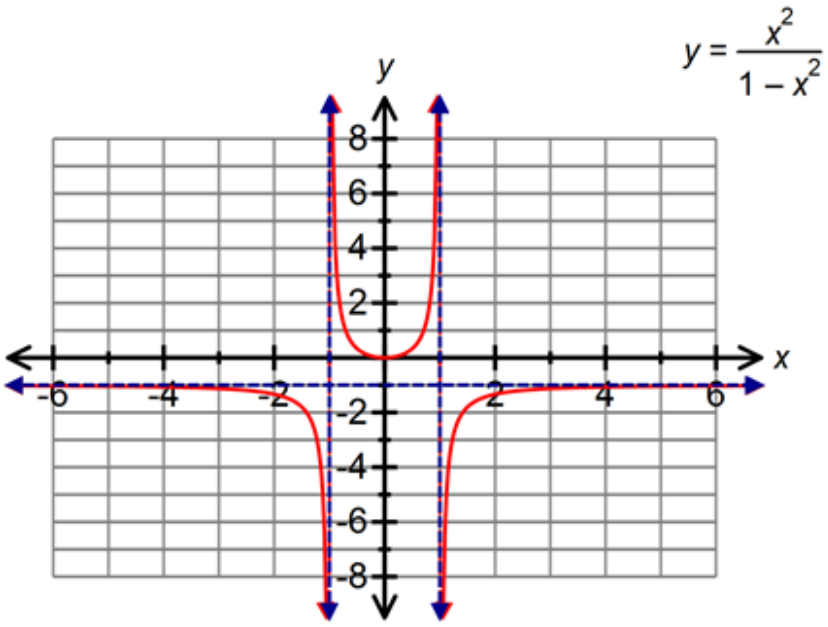
Use Derivatives to Sketch the Graph of a Rational Function

Idea:

(1) use the first derivative to identify critical numbers, relative extrema and intervals of increase and decrease.

(2) use the second derivative to identify hypercritical numbers, points of inflection and intervals of concavity.

(3) Identify intercepts, asymptotes, points of discontinuity, and the domain.



Example 1: $f(x) = \dfrac{x^2}{1-x^2}$

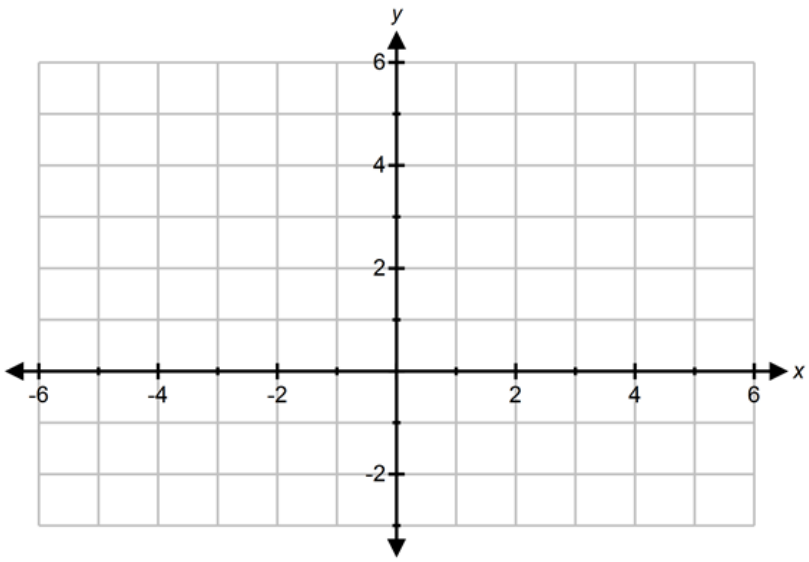
Determine where the graph is increasing, decreasing, any relative maximum and minimum points, where the graph is concave upward and concave downward, and any inflection points. Sketch a labelled graph including the intercepts.

$f'(x) = \dfrac{2x}{(1-x^2)^2}$ and $f''(x) = \dfrac{2+6x^2}{(1-x^2)^3}$

Example 2: $f(x) = \dfrac{x^2(x+2)}{(x-1)^3}$

Determine where the graph is increasing, decreasing, any relative maximum and minimum points, where the graph is concave upward and concave downward, and any inflection points. Sketch a labelled graph including the intercepts.

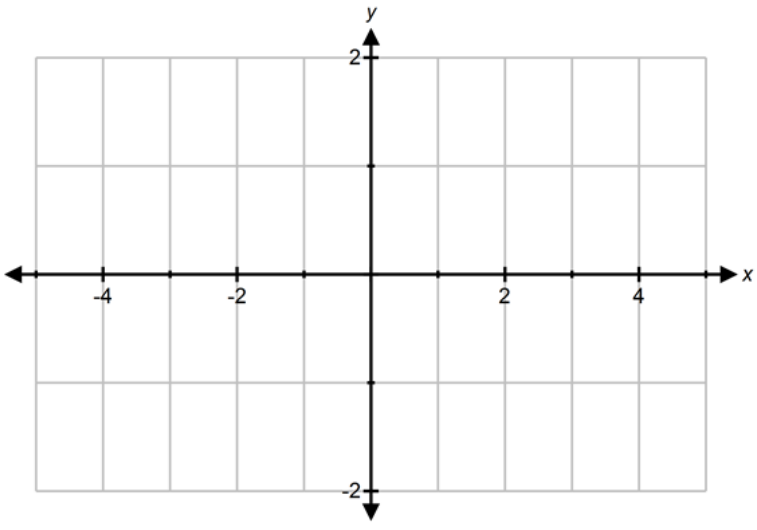
$f'(x) = \dfrac{-x(5x+4)}{(x-1)^4}$ $\qquad$ $f''(x) = \dfrac{2(5x+1)(x+2)}{(x-1)^5}$



Example 3: $y = \dfrac{6}{x^2 + 3}$

Determine where the graph is increasing, decreasing, any relative maximum and minimum points, where the graph is concave upward and concave downward, and any inflection points. Sketch a labelled graph including the intercepts.

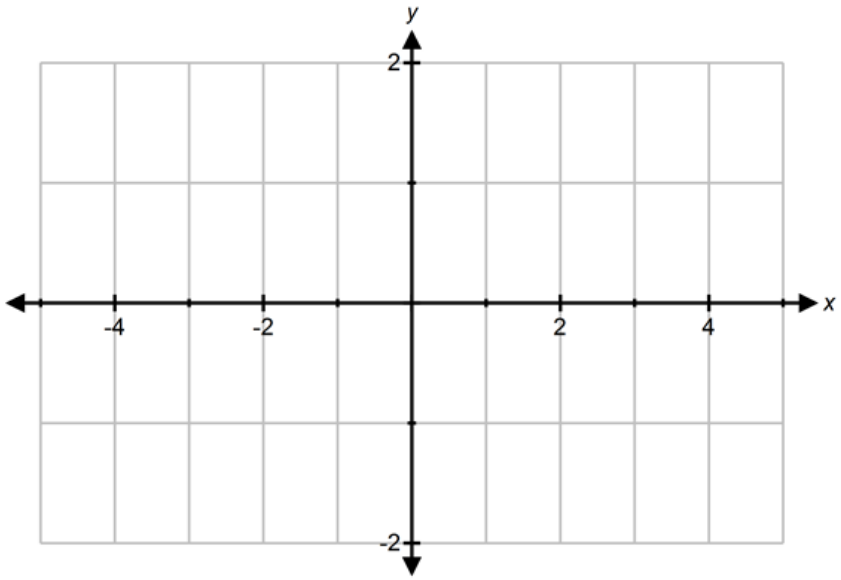
$y' = \dfrac{-12x}{(x^2 + 3)^2}$ and $y'' = \dfrac{-12(-3x^2 + 3)}{(x^2 + 3)^3}$



Example 4: $y = \dfrac{x+1}{(x-1)^2}$

Determine where the graph is increasing, decreasing, any relative maximum and minimum points, where the graph is concave upward and concave downward, and any inflection points. Sketch a labelled graph including the intercepts.

$y' = \dfrac{-(x+3)}{(x-1)^3}$ and $y'' = \dfrac{2x+10}{(x-1)^4}$



Example 5: $y = \dfrac{x^2}{(x-2)^2}$

Determine where the graph is increasing, decreasing, any relative maximum and minimum points, where the graph is concave upward and concave downward, and any inflection points. Sketch a labelled graph including the intercepts.

Note:

$y' = \dfrac{-4x}{(x-2)^3}$ and $y'' = \dfrac{8x+8}{(x-2)^4}$