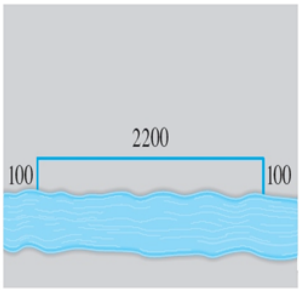
Lesson 5.4: Optimization Problems

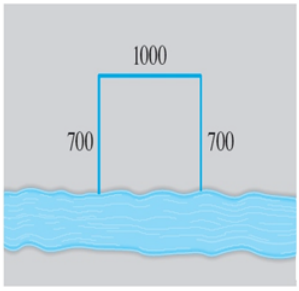
↓ quantity that is to be maximized or minimized

Example:

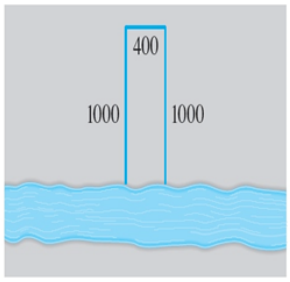
A farmer has 2400 ft of fencing and wants to fence a rectangular field that borders a straight river. He needs no fence along the river. What are the dimensions of the filed that has the largest area?



Area = $100 \cdot 2200 = 220{,}000\ \text{ft}^2$



Area = $700 \cdot 1000 = 700{,}000\ \text{ft}^2$



Area = $1000 \cdot 400 = 400{,}000\ \text{ft}^2$

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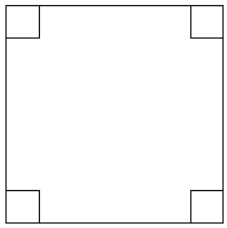
Example 1

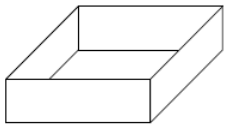
Module 3.5

An open-top box is to be made by cutting congruent squares of side length x from the corners of a 20-by-25 inch sheet of tin and bending up the sides.



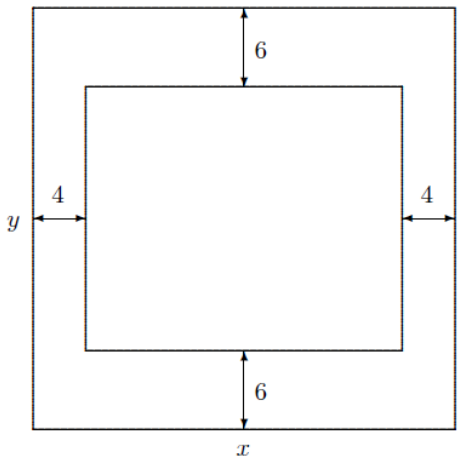
(i) How large should the squares be to make the box hold as much as possible?

(ii) What is the resulting maximum volume?



Example 2

The top and bottom margins of a poster are each 6 cm and the side margins are each 4 cm. The area of the printed material on the poster (which must fall within the margins) is 384 cm^2.What are the dimensions of the poster with the smallest area?



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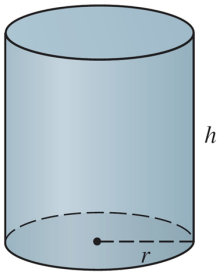
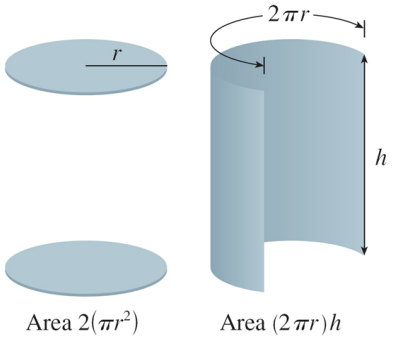
A rectangular storage container with an open top is to have a volume of 10 m^3. The length of its base is twice the width. Material for the base costs $10 per square meter. Material for the sides cost $6 per square meter. Find the cost of materials for the cheapest such container.

Example 4

A cylindrical can is to be made to hold 1L of oil. Find the dimensions that will minimize the cost of the metal to manufacture the can.

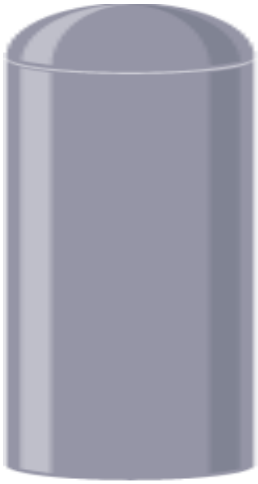
$1L = 1000\,cm^3$





Example 5

A special shipping tube consists of a cylinder which is closed on the bottom but capped by a hemisphere on the top. Its total volume is 1 cubic metre. The material which forms the cylindrical part of the tube (including the bottom) costs $2.00 per square metre, while the material to make the hemispherical top costs $3.50 per square metre. What is the cost of the cheapest such tube?



Example 6

Two posts, one 12 feet high and the other 28 feet high, stand 30 feet apart. They are to be stayed by two wires, attached to a single stake, running from ground level to the top of each post. Where should the stake be placed to use the least amount of wire?