Section 7.2: Derivatives of General Exponential and Logarithmic Functions

$y = a^x$ $\qquad$ $y = \log_a x$

(i) Differentiate $y = a^x$

Review: $\frac{d}{dx}(e^x) = e^x$

$y = a^x$ $\qquad$ Apply "ln" to both sides; use power rule for logs; use implicit differentiation

Chain Rule: $\frac{d}{dx}(a^u) = a^u \ln a \cdot \frac{du}{dx}$

(ii) Differentiate $y = \log_a x$

Review: $\frac{d}{dx}(\ln x) = \frac{1}{x}$

$y = \log_a x$ change to exponential form and use implicit differentiation

Chain Rule: $\frac{d}{dx}(\log_a u) = \frac{1}{u \ln a} \cdot \frac{du}{dx}$

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Summary

↳ Derivatives of Exponential and Logarithmic Functions

Exponential Functions	
$y = e^x$	$\frac{dy}{dx} = e^x$
$y = a^x$	$\frac{dy}{dx} = a^x \ln a$

Logarithmic Functions	
$y = \ln x$	$\frac{dy}{dx} = \frac{1}{x}$
$y = \log_a x$	$\frac{dy}{dx} = \frac{1}{x \ln a}$

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Example 1

Determine y' for the following:

(a) $y = 3^{\sin x}$

(b) $y = (\ln 2)^x$

(c) $y = \tan(3^{x^2})$

(d) $y = x\sin(2^x)$

(e) $y = 2^{\sec^4(\sqrt{x})}$

(f) $y = \pi^{x-2} + e^{\pi-2}$

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Example 2: Differentiate

(a) $y = \log_2(\sin x + \cos x)$

(b) $y = \log_3 \sqrt{x^3 + x^2}$

(c) $y = \log(\log(2x^3))$

(d) $y = 2^{\log x} - \log(2^x)$

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(e) $y = \log_3(\sin x^2)$

(f) $y = 2^{x^3} \log(x^2)$

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Application Problems

↳ Tangent and Normal Lines

a) Write the equation of a tangent and normal line that touch the curve $f(x) = 6 - e^x$ at $x = \ln 2$.

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b) Write the equation of a tangent line that touch the curve
$f(x) = x\ln x - x$ at $x = 1$.

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