Section 7.3 Logarithmic Differentiation

- to simplify a complicated derivative involving products, quotients and powers using the rules of logarithms.

  Example: $y = \dfrac{(2x+1)^3(x^3+x)^4}{(x^2+1)^2}$

- to deal with functions that have variables in the base as well as in the exponent.

  Example: $y = x^{\cos x}$

Example 1

Determine the derivative using logarithmic differentiation

$$y = x^x$$

Example 2

Determine the derivative using logarithmic differentiation

$$y = x^{\cos(x)}$$

Example 3

Determine the derivative using logarithmic differentiation

$$y = \frac{(2x+1)^3(x^3+x)^4}{(x^2+1)^2}$$

Idea:

$$y = \frac{(f(x))^3(g(x))^4}{(h(x))^2} \quad \xrightarrow{\text{rewrite}} \quad \ln y = 3\ln(f(x)) + 4\ln(g(x)) - 2\ln(h(x))$$

Three chain rules inside a product inside a quotient

Implicit differentiation with 3 quick chain rules

Example 4

Apply natural logarithms to prepare the function for logarithmic differentiation.

$$y = 4^x e^{2x} \tan(5x - 1)$$

Example 5

Determine the derivative using logarithmic differentiation

$$y = \sqrt[3]{\frac{(x^2+1)\sin^2 x}{(x^3+3)\cos^4 x}}$$

Example 6

Differentiate the following:

(a) $y = \dfrac{x^{\frac{2}{3}}\sqrt{x^2+2}}{(x^3(6x+1))^3}$

(b) $y = \dfrac{(x^3+1)^4 \sin^2 x}{x^{\frac{1}{3}}}$

(c) $y = (\sin x)^{\ln x}$

(d) $y = (\tan x)^{\frac{1}{x}}$

(e) Show $\dfrac{d}{dx}(\cos x)^{\tan x} = (\cos x)^{\tan x}[\sec^2 x \ln(\cos x) - \tan^2 x]$

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