Calculus 3208

Related Rates

1. $V = \frac{4}{3}\pi r^3$

$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

$\frac{dr}{dt} = -0.15 \text{ cm/min}$ when $r = 6\text{cm}$

$\frac{dV}{dt} = 4\pi (6)^2 (-0.15 \text{ cm/min})$

$\frac{dV}{dt} = -21.6\pi \text{ cm}^3/\text{min}$

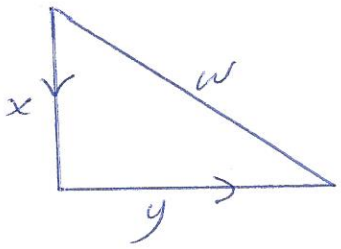
$A = 4\pi r^2$

$\frac{dA}{dt} = 8\pi r \frac{dr}{dt}$

$\frac{dA}{dt} = 8\pi (6)(-0.15)$

$\frac{dA}{dt} = -7.2\pi \text{ cm}^2/\text{min}$

2.



$w^2 = x^2 + y^2$

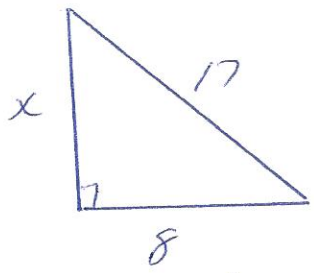
$2w\frac{dw}{dt} = 2x\frac{dx}{dt} + 2y\frac{dy}{dx}$

$\frac{dw}{dt} = 0 \quad \frac{dy}{dt} = 2 \frac{ft}{sec}$

$0 = 2x\frac{dx}{dt} + 2y\frac{dy}{dt}$

$\frac{dx}{dt} = ?$

$-x\frac{dx}{dt} = y\frac{dy}{dt}$



$\frac{dx}{dt} = -\frac{y}{x}\frac{dy}{dt}$

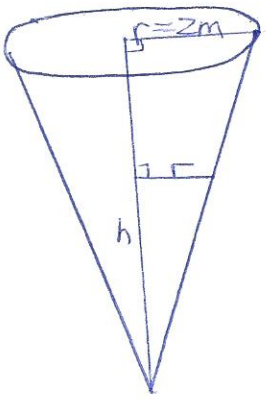
$17^2 = x^2 + 8^2$

$x = 15$

$\frac{dx}{dt} = -\frac{8}{15}(2)$

$\frac{dx}{dt} = -\frac{16}{15} \text{ ft/sec}$

3.



$\frac{2}{6} = \frac{r}{h}$

$2h = 6r$

$\frac{2h}{6} = r$

$\frac{h}{3} = r$

$\frac{dV}{dt} = -1 m^3/min$

$\frac{dh}{dt} = ?$ $(h = 4m)$

$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi \left(\frac{h}{3}\right)^2 h$

$V = \frac{1}{3}\pi \left(\frac{h^2}{9}\right) h$

$V = \frac{\pi}{27} h^3$

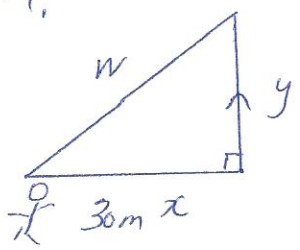
$\frac{dV}{dt} = \frac{\pi}{9} h^2 \frac{dh}{dt}$

$-1 = \frac{\pi}{9}(4)^2 \frac{dh}{dt}$

$-1 = \frac{16\pi}{9} \frac{dh}{dt}$

$-\frac{9}{16\pi} m/min = \frac{dh}{dt}$

4.



$\frac{dy}{dt} = 2 m/sec$

$\frac{dx}{dt} = 0$

$\frac{dw}{dt} = ?$

$w^2 = x^2 + y^2$

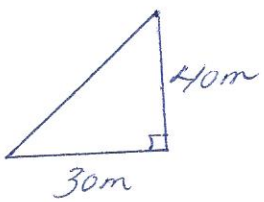
$2w \frac{dw}{dt} = 2x \frac{dx}{dt} + 2y \frac{dy}{dt}$

$w \frac{dw}{dt} = y \frac{dy}{dt}$

$50 \frac{dw}{dt} = 40(2)$

$\frac{dw}{dt} = \frac{80}{50}$

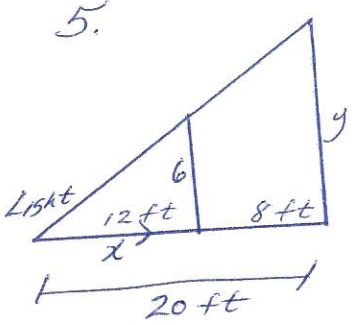
$\frac{dw}{dt} = 8/5 \; m/sec$



$w^2 = 30^2 + 40^2$

$w = 50$

5.



$\frac{dx}{dt} = 2.5 \text{ ft/sec}$

$\frac{dy}{dt} = ?$

$\frac{20}{x} = \frac{y}{6}$

$120 = xy$

$\frac{120}{x} = y$

$120x^{-1} = y$

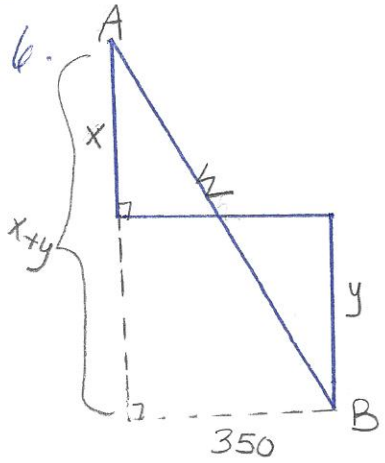
$-120x^{-2}\frac{dx}{dt} = \frac{dy}{dt}$

$-\frac{120}{x^2}\frac{dx}{dt} = \frac{dy}{dt}$

$-\frac{120}{(12)^2}(2.5) = \frac{dy}{dt}$

$-2.08 \text{ ft/sec} = \frac{dy}{dt}$ (decreasing shadow)

6.



$w^2 = (x+y)^2 + 350^2$

$2w\frac{dw}{dt} = 2(x+y)\left(\frac{dx}{dt} + \frac{dy}{dt}\right) + 0$

$10745.7015\frac{dw}{dt} = (7500 + 3240)(5+3)$

$\frac{dw}{dt} = 7.9958 \text{ m/sec}$

$\frac{dx}{dt} = 5 \text{ m/sec}$ $\frac{dy}{dt} = 3 \text{ m/sec}$

A ⇒ after 25min = 1500sec $x = 1500(5)$
$x = 7500\text{m}$

B ⇒ after 18min = 1080sec $y = 1080(3)$
$y = 3240\text{m}$

$w = 10745.7015\text{m}$ (apart)

7.

$\frac{dr}{dt} = -0.1$ cm/sec

$\frac{dV}{dt} = ?$

$V = 800\pi$ cm^3

height is fixed $h = 6$cm

$\frac{dh}{dt} = 0$

$V = \frac{1}{3}\pi r^2 h$

$800\pi = \frac{1}{3}\pi r^2 (6)$

$800\pi = 2\pi r^2$

$400 = r^2$

$20 = r$

$V = \frac{1}{3}\pi r^2 h$

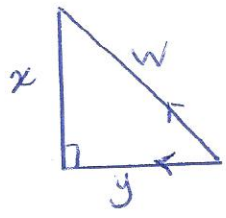
$\frac{dV}{dt} = \frac{1}{3}\pi \left[r^2 \cdot \frac{dh}{dt} + 2rh \frac{dr}{dt} \right]$

$\frac{dV}{dt} = \frac{1}{3}\pi \left[0 + 2rh \frac{dr}{dt} \right]$

$\frac{dV}{dt} = \frac{1}{3}\pi \left[2(20)(6)(-0.1) \right]$

$\frac{dV}{dt} = -80\pi$ cm^3/sec

8.



$\frac{dw}{dt} = -1$ M/sec

$\frac{dx}{dt} = 0$

$\frac{dy}{dt} = ?$ $y = 8$

$x^2 + y^2 = w^2$

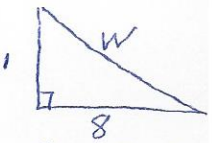
$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2w\frac{dw}{dt}$

($x \to 0$)

$y\frac{dy}{dt} = w\frac{dw}{dt}$

$\frac{dy}{dt} = \frac{w}{y}\frac{dw}{dt}$

$\frac{dy}{dt} = \frac{\sqrt{65}}{8}(-1 \text{M/sec}) = -1.01$ M/s

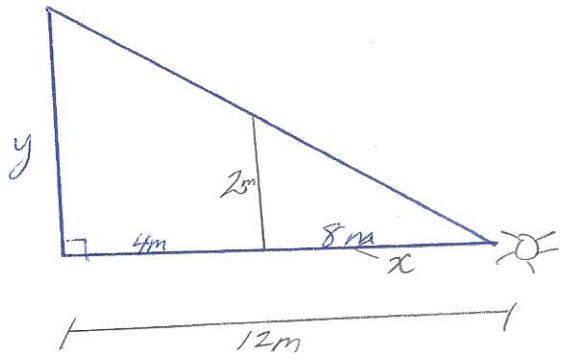


$w^2 = 8^2 + 1^2$

$w^2 = 65$

$w = \sqrt{65}$

9.



$\frac{dx}{dt} = 1.6 \text{ m/sec}$

$\frac{dy}{dt} = ?$

$$\frac{y}{2} = \frac{12}{x}$$

$$24 = xy$$

$$\frac{24}{x} = y$$

$$24x^{-1} = y$$

$$-24x^{-2}\frac{dx}{dt} = \frac{dy}{dt}$$

$$-\frac{24}{x^2}\frac{dx}{dt} = \frac{dy}{dt}$$

$$-\frac{24}{(8)^2}(1.6) = \frac{dy}{dt}$$

$$-0.6 \text{ m/sec} = \frac{dy}{dt}$$

10. Sphere $V = \frac{4}{3}\pi r^3$

$\frac{dV}{dt} = 648 \frac{cm^3}{sec}$

$\frac{dr}{dt} = ?$

$V = 972\pi cm^3$

$972\pi = \frac{4}{3}\pi r^3$

$972\pi \cdot \frac{3}{4\pi} = r^3$

$729 = r^3$

$9 = r$

$\frac{dV}{dt} = \frac{4}{3}\pi\left[3r^2 \frac{dr}{dt}\right]$

$\frac{dV}{dt} = 12\pi r^2 \frac{dr}{dt}$

$648 = 12\pi (9)^2 \frac{dr}{dt}$

$648 = 972\pi \frac{dr}{dt}$

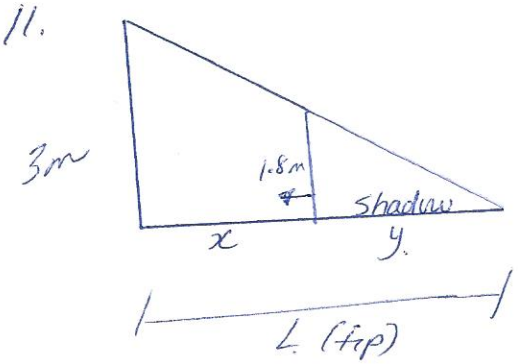
$\frac{648}{972\pi} = \frac{dr}{dt}$

$\frac{2}{3\pi} = \frac{dr}{dt}$

$0.212 \frac{cm}{sec} = \frac{dr}{dt}$

180cm = 1.8m MAN

11.



$\frac{dx}{dt} = -2.4$ m/s

$\frac{dy}{dt}$ → length of shadow changing

$\frac{dL}{dt}$ → tip of shadow changing

a) Find $\frac{dy}{dt}$: ratios $\frac{y}{x+y} = \frac{1.8}{3}$

$3y = 1.8x + 1.8y$

$1.2y = 1.8x$

$y = 1.5x$

$\frac{dy}{dt} = 1.5\frac{dx}{dt}$

$\frac{dy}{dt} = 1.5(-2.4)$

$\frac{dy}{dt} = -3.6$ m/sec

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b) Find $\frac{dL}{dt}$ ratios $\frac{y}{L} = \frac{1.8}{3}$

$3.y = 1.8L$

$\frac{3}{1.8}y = L$

$\frac{3}{1.8}\frac{dy}{dt} = \frac{dL}{dt}$

$\frac{3}{1.8}(-3.6) = \frac{dL}{dt}$

-6 m/sec $= \frac{dL}{dt}$