Calculus 3208 **Name: ___________________**

Review Sheet: Applications of Derivatives

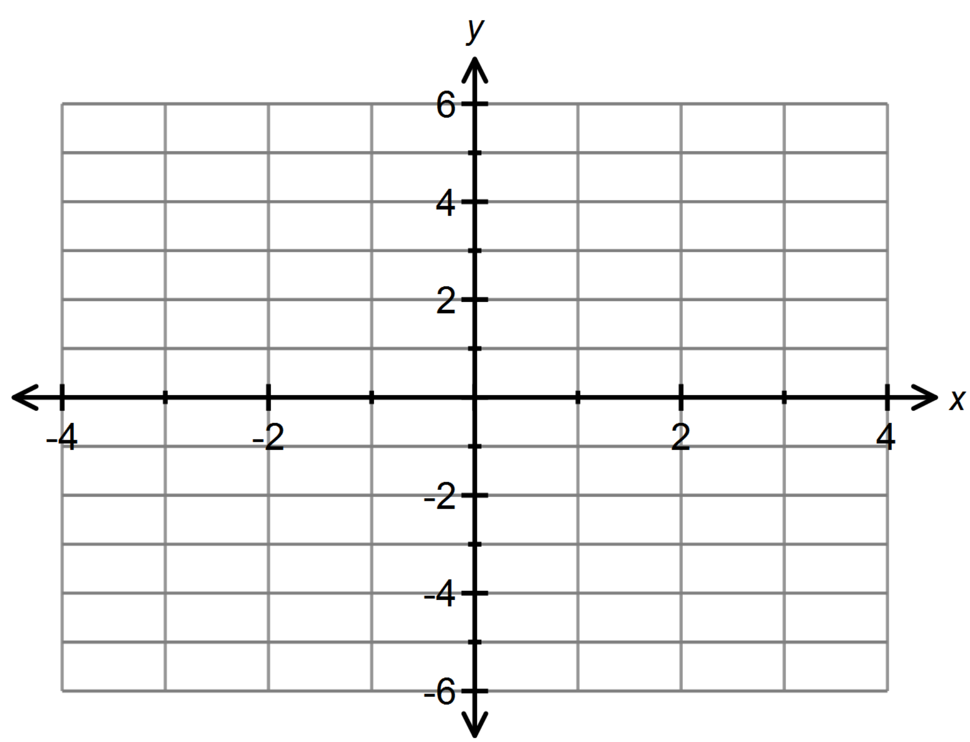
1. Given the following function $f(x) = 4 - 3x^2 + x^3$, answer the following questions:

   (a) Determine the *x* and y-intercepts of the graph of $f(x)$, if any.

   (b) Determine the intervals on which $f(x)$ is increasing or decreasing and classify any relative (local) extrema.

   (c) Determine the intervals on which $f(x)$ is concave upward or concave downward and identify any inflection points.

   (d) Sketch the graph of $f(x)$. Label any inflection points and extrema.



2. Given the following function with the indicated derivatives, answer the following questions:

$$f(x) = \frac{x^2+1}{x^2-1}; \quad f'(x) = \frac{-4x}{(x^2-1)^2}; \quad f''(x) = \frac{4(3x^2+1)}{(x^2-1)^3}$$

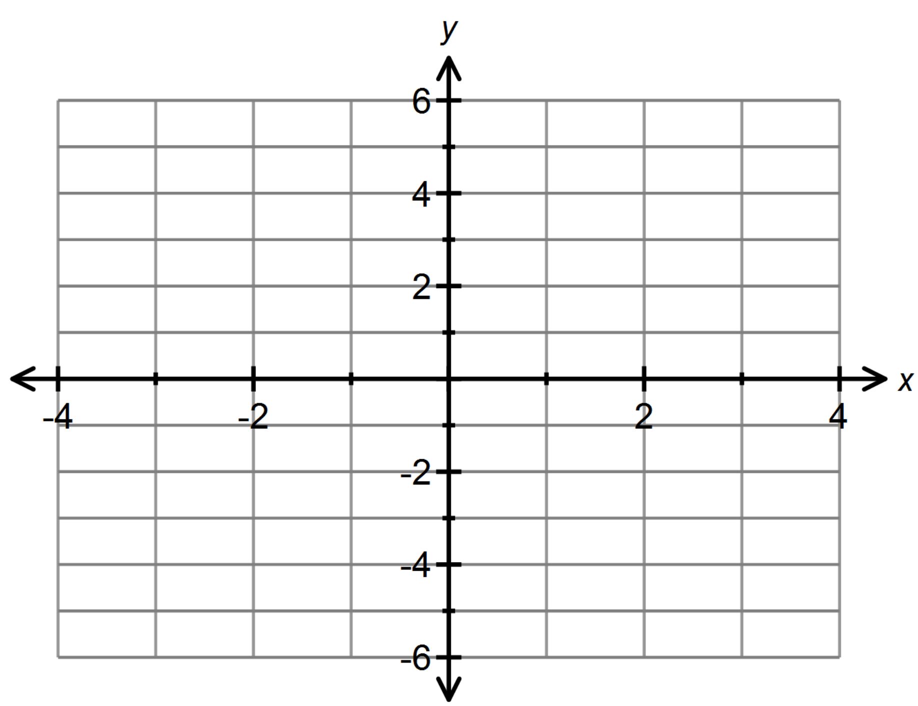
(a) Determine the vertical and horizontal asymptotes of $f(x)$, if any.

(b) Determine the *x* and y-intercepts of the graph of $f(x)$, if any.

(c) Determine the intervals on which $f(x)$ is increasing or decreasing and classify any relative (local) extrema.

(d) Determine the intervals on which $f(x)$ is concave upward or concave downward and identify any inflection points.

(e) Sketch the graph of $f(x)$. Label any inflection points and extrema.



3. A snowball is melting such that the radius is decreasing by 0.15cm/min. How fast is the volume changing when the radius is 6 cm? $\left[V = \frac{4}{3}\pi r^3\right]$

4. Edwin and Stella part ways at a crossroads. Stella moodily runs to the west, moving at a constant rate of 2 metres per second. Edwin glumly watches her vanish into the distance for 5 minutes before he turns to the north and heads in that direction at a constant rate of 1.5 metres per second. Determine how quickly the distance between them is increasing **15 minutes** after Stella's departure.

5. Two cars start out 500 miles apart. Car A is to the west of Car B and starts driving to the east (*i.e.* towards Car B) at 35 mph and at the same time Car B starts driving south at 50 mph. After 3 hours of driving at what rate is the distance between the two cars changing? Is it increasing or decreasing?

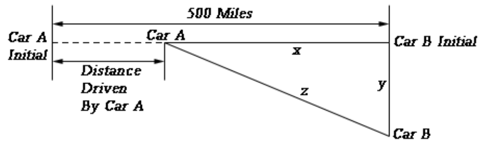
6. Sand falling from a chute forms a conical pile whose height is always equal to 4/3 of the radius of the base. How fast is the volume changing when the radius of the base is 36 inches and is increasing at a rate of 3 in/min? $\left[V = \frac{1}{3}\pi r^2 h\right]$

7. An ice sculpture in the shape of an inverted cone is melting such that its height shrinks at a rate of $\frac{1}{3}$ metres per hour but its radius remains constant. How fast is the cone's voulume decreasing when its height is 1 metre and its volume is 3π cubic metres?

8. A box with a square base and open top is to be made with $1200\,cm^2$ of cardboard. Determine the dimensions which will give the largest possible volume of the box.

9. We want to construct a box whose base length is 3 times the base width. The material used to build the top and bottom cost $10/ft^2 and the material used to build the sides cost $6/ft^2. If the box must have a volume of 50 ft^3 , determine the dimensions that will minimize the cost to build the box.

10. A printer needs to make a poster that will have a total area of 200 in^2 and will have 1 inch margins on the sides, a 2 inch margin on the top and a 1.5 inch margin on the bottom. What dimensions will give the largest printed area (the area of the poster with the margins taken out)?



n this figure y represents the distance driven by Car B and x represents the distance separating Car A from Car B's initial position and z represents the distance separating the two cars. After 3 hours driving time with have the following values of x and y.

$$x = 500 - 35(3) = 395 \qquad y = 50(3) = 150$$

Ve can use the Pythagorean theorem to find z at this time as follows,

$$z^2 = 395^2 + 150^2 = 178525 \qquad \Rightarrow \qquad z = \sqrt{178525} = 422.5222$$

Vow, to answer this question we will need to determine z' given that $x' = -35$ and $y' = 50$. Do you agree with the signs on the two given rates? Remember that a rate is negative if the quantity is decreasing and positive if the quantity is increasing.

Ve can again use the Pythagorean theorem here. First, write it down and the remember that x, y, and z are all changing with time and so differentiate the equation using **Implicit Differentiation**.

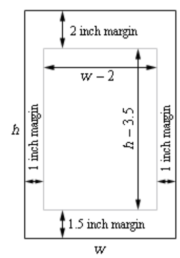
$$z^2 = x^2 + y^2 \qquad \Rightarrow \qquad 2zz' = 2xx' + 2yy'$$

Finally, all we need to do is cancel a two from everything, plug in for the known quantities and solve for z'.

$$z'(422.5222) = (395)(-35) + (150)(50) \qquad \Rightarrow \qquad z' = \frac{-6325}{422.5222} = -14.9696$$

So, after three hours the distance between them is decreasing at a rate of 14.9696 mph.

Here is a sketch of the poster and we can see that once we've taken the margins into account the width of the printed area is $w-2$ and the height of the printer area is $h-3.5$.



Here are the equations that we'll be working with.

$$\text{Maximize}: A = (w-2)(h-3.5)$$

$$\text{Constraint}: 200 = wh$$

Solving the constraint for h and plugging into the equation for the printed area gives,

$$A(w) = (w-2)\left(\frac{200}{w} - 3.5\right) = 207 - 3.5w - \frac{400}{w}$$

The first and second derivatives are,

$$A'(w) = -3.5 + \frac{400}{w^2} = \frac{400 - 3.5w^2}{w^2} \qquad A''(w) = -\frac{800}{w^3}$$

From the first derivative we have the following three critical points.

$$w = 0 \qquad w = \pm\sqrt{\tfrac{400}{3.5}} = \pm 10.6904$$

However, since we're dealing with the dimensions of a piece of paper we know that we must have $w > 0$ and so only 10.6904 will make sense.