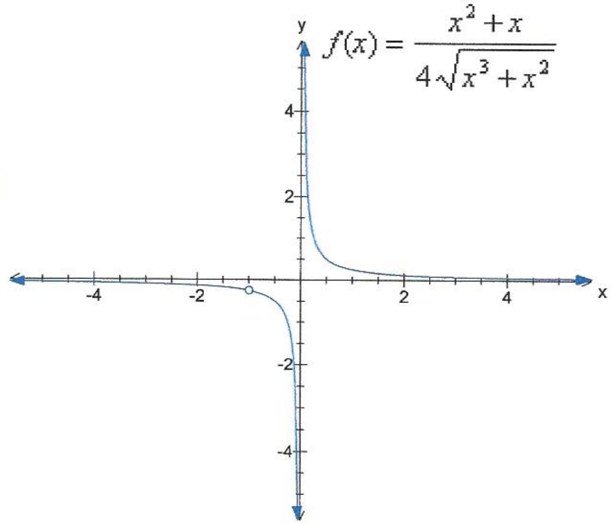
1. Use the graph of the function to state the value of $\lim_{x \to 0} f(x)$, if it exists.

A. ∞
B. $\frac{1}{4}$
C. $-\infty$
D. does not exist

$\lim_{x \to 0^+} f(x) = \infty$

$\lim_{x \to 0^-} f(x) = -\infty$



2. Evaluate the limit: $\lim_{h \to -1} \left(h^4 - h^3 - 4h + 6\right)$.

A. 12
B. –12
C. 6
D. –1

$(1) - (-1) - 4(-1) + 6$
$= 12$

3. Evaluate the limit: $\lim_{x \to 9} \frac{x^2 - 81}{\sqrt{x} - 3} \left(\frac{0}{0}\right)$

A. 48
B. 81
C. 96
D. 108

$\lim_{x \to 9} \frac{(x+9)(x-9)}{\sqrt{x} - 3} \left(\frac{\sqrt{x}+3}{\sqrt{x}+3}\right)$

$\lim_{x \to 9} \frac{(x+9)\cancel{(x-9)}(\sqrt{x}+3)}{\cancel{(x-9)}} = 18 \times 6$

4. Determine the limit $\lim_{x \to 3} \frac{x^2 - x - 6}{x - 3}$, if it exists.

A. 2
B. 5
C. 3
D. Does not exist

$\lim_{x \to 3} \frac{\cancel{(x-3)}(x+2)}{\cancel{(x-3)}} = 5$

5. Evaluate the limit $\lim_{x \to 2} \frac{x-1}{x^2-4x+7}$.

$\frac{2-1}{4-8+7} = \frac{1}{3}$

A. $\frac{1}{7}$

B. $\frac{1}{3}$

C. -1

D. 1

6. Determine the limit $\lim_{x \to 0} \frac{\sqrt{x+14}-\sqrt{14}}{x}$, if it exists.

$\frac{0}{0}$

$\lim_{x \to 0} \frac{\sqrt{x+14}-\sqrt{14}}{x} \cdot \left(\frac{\sqrt{x+14}+\sqrt{14}}{\sqrt{x+14}+\sqrt{14}}\right)$

$\lim_{x \to 0} \frac{(x+14)-14}{x(\sqrt{x+14}+\sqrt{14})} = \lim_{x \to 0} \frac{\cancel{x}}{\cancel{x}(\sqrt{x+14}+\sqrt{14})}$

$= \frac{1}{2\sqrt{14}} \cdot \frac{\sqrt{14}}{\sqrt{14}} = \frac{\sqrt{14}}{28}$

A. $\frac{\sqrt{14}}{2}$

B. $\frac{\sqrt{14}}{14}$

C. $\frac{\sqrt{14}}{28}$

D. Does not exist

7. Evaluate the $\lim_{x \to 4} \frac{|x-4|}{x-4}$.

$\lim_{x \to 4^+} \frac{(x-4)}{x-4} = 1$

$\lim_{x \to 4^-} \frac{-(x-4)}{x-4} = -1$

A. 1

B. -1

C. 0

D. Does not exist

8. Let $f(x) = \begin{cases} \frac{x^2-36}{x-6}, & \text{if } x \neq 6 \\ 12, & \text{if } x = 6 \end{cases}$ $\qquad \frac{(x-6)(x+6)}{x-6}$

Which of the following statement(s) is (are) true?

I. f is differentiable at $x = 6$.

II. $\lim_{x \to 6} f(x)$ *exists*

III. f is continuous at $x = 6$.

$f(6) = 12$

$\lim_{x \to 6} f(x) = 12$ continuous

A. I only

B. II only

C. I and II only

D. I, II, II

Differentiable

$f(6)$ defined $= 12$

$f'(6) = \lim_{x \to 6} \frac{f(x) - f(6)}{x-6} = \lim_{x \to 6} \frac{x+6-12}{x-6} = 1$ exists

or $\lim_{x \to 6} f'(x) = \lim_{x \to 6} 1 = 1$

9. Choose an equation from the following that expresses the fact that a function f is continuous at the number 6.

A. $\lim_{x\to 0} f(x) = 6$

B. $\lim_{x\to 0} f(x) = f(6)$

C. $\lim_{x\to 6^-} f(x) = -\infty$

D. $\lim_{x\to 6} f(x) = f(6)$

10. If f and g are continuous functions with $f(9) = 2$ and $\lim_{x\to 9}\left[2f(x) - g(x)\right] = 9$, find $g(9)$.

A. $g(9) = 13$
B. $g(9) = 11$
C. $g(9) = 4$
D. $g(9) = -5$

$\lim_{x\to 9} 2f(x) - \lim_{x\to 9} g(x)$

$2\lim_{x\to 9} f(x) - \lim_{x\to 9} g(x) = 9$

$2(2) - \lim_{x\to 9} g(x) = 9$

$g(9) = -5$

11. Determine where f is discontinuous.

$$f(x) = \begin{cases} \sqrt{-x} & \text{if } x < 0 \\ 11 - x & \text{if } 0 \le x < 11 \\ (11-x)^2 & \text{if } x \ge 11 \end{cases}$$

A. 0 only
B. 0 and 11
C. 0 and − 11
D. 11 only
E. −11 only

Check $x=0$

$f(0) = 11$

$\left.\begin{array}{l}\lim_{x\to 0^+} (11-x) = 11 \\ \lim_{x\to 0^-} \sqrt{-x} = 0\end{array}\right\}$ limit DNE

check $x=11$

$f(11) = (11-x)^2 = 0$

$\left.\begin{array}{l}\lim_{x\to 11^+} (11-x)^2 = 0 \\ \lim_{x\to 11^-} (11-x) = 0\end{array}\right\}$ limit exists

12. For what value of the constant c is the function f continuous on $(-\infty, \infty)$?

$$f(x) = \begin{cases} cx + 7 & \text{for } x \le 2 \\ cx^2 - 5 & \text{for } x > 2 \end{cases}$$

A. $c = 1$
B. $c = 2$
C. $c = -2$
D. $c = -6$
E. $c = 6$

function is different at $x=2$

$\lim_{x\to 2^+} f(x) = \lim_{x\to 2^-} f(x) = f(2)$

$\left.\begin{array}{l}\lim_{x\to 2^+} (cx^2-5) = 4c-5 \\ \lim_{x\to 2^-} (cx+7) = 2c+7\end{array}\right\}$

$4c-5 = 2c+7$
$2c = 12$
$c = 6$

$f(2) = 2c+7$
$f(2) = 2(6)+7 = 19$

13. Determine the limit: $\lim_{x\to\infty}\frac{7x-2}{5x+5}$

$\lim_{x\to\infty} \frac{\frac{7x}{x}-\frac{2}{x}}{\frac{5x}{x}+\frac{5}{x}} = \frac{7}{5}$

A. $\frac{5}{2}$

B. $\frac{7}{5}$

C. $\frac{2}{7}$

D. $\frac{5}{7}$

E. $\frac{2}{5}$

14. Find the limit. $\lim_{x\to\infty}(4x^2-7x^4)$ larger even larger

A. ∞

B. $-\infty$

C. 6

D. -6

E. 5

15. Determine the vertical asymptotes of the function: $y=\frac{4x^2+1}{3x-4x^2}$ $= \frac{4x^2+1}{x(3-4x)}$

$x=0$ $\quad 3-4x=0$

$3=4x$

$\frac{3}{4}=x$

A. $x=0,\frac{3}{4}$

B. $x=\frac{3}{4}$

C. $x=0,\frac{4}{3}$

D. $x=\frac{4}{3}$

16. What is the horizontal asymptote of the graph of the function: $f(x)=\frac{x^2+3x-2}{2x^2+4x-5}$

$\lim_{x\to\pm\infty} \frac{\frac{x^2}{x^2}+\frac{3x}{x^2}-\frac{2}{x^2}}{\frac{2x^2}{x^2}+\frac{4x}{x^2}-\frac{5}{x^2}} = \frac{1}{2}$

A. $y=-2$

B. $y=\frac{1}{2}$

C. $y=-\frac{1}{2}$

D. $y=2$

17. What is the oblique asymptote for the graph of: $f(x)=\frac{3x^2-x+6}{x+2}$?

A. $y=3x-7$

B. $y=x-3$

C. $y=3x+5$

D. $y=x+1$

$$\begin{array}{r|rrr} -2 & 3 & -1 & 6 \\ & & -6 & 14 \\ \hline & 3 & -7 & 20 \end{array}$$

$y=3x-7$

18. Determine the $\lim_{x\to a}\frac{x^2-a^2}{x^4-a^4}$ where $a\neq 0$.

A. $\frac{1}{a^2}$

B. $\frac{1}{2a^2}$

C. $\frac{1}{6a^2}$

D. 0

$\lim_{x\to a} \frac{(x-a)(x+a)}{(x^2-a^2)(x^2+a^2)}$

$\lim_{x\to a} \frac{(x-a)(x+a)}{(x-a)(x+a)(x^2+a^2)} = \frac{1}{2a^2}$

derivative

19. What is the instantaneous rate of change at $x = 2$ of the function f given by $f(x) = \dfrac{x^2 - 1}{x - 1}$.

A. -2
B. 1
C. -1
D. 2

$f'(2) = \dfrac{(x-1)(2x) - (x^2-1)(1)}{(x-1)^2}$

$= \dfrac{2x^2 - 2x - x^2 + 1}{(x-1)^2} = \dfrac{4-4+1}{1} = 1$

20. At what point on the graph of $y = \frac{1}{2}x^2$ is the tangent line parallel to the line $2x - 4y = 3$?

A. $\left(\frac{1}{2}, -\frac{1}{2}\right)$
B. $\left(\frac{1}{2}, \frac{1}{8}\right)$
C. $\left(1, -\frac{1}{4}\right)$
D. $\left(1, \frac{1}{2}\right)$.

$y' = 1x$
$y' = m$
$\frac{1}{2} = x$

$-4y = -2x + 3$
$y = \frac{1}{2}x - 3/4$
$m = \frac{1}{2}$

$y = \frac{1}{2}(\frac{1}{2})^2$
$y = 1/8$

21. If $h(x) = f(g(x))$ with $g(x) = x^2 + 4x$, $f'(12) = 2$, $f'(8) = 3$ and $f'(2) = 4$ then $h'(2) = ?$

A. 16
B. 24
C. 36
D. 48

$h'(x) = f'(g(x)) \cdot g'(x)$
$h'(2) = f'(g(2)) \cdot g'(2)$
$h'(2) = f'(12) \cdot (8)$
$h'(2) = (2)(8)$

$g'(x) = 2x + 4$
$g'(2) = 2(2) + 4 = 8$
$g(2) = 4 + 8 = 12$

22. What is the equation of the normal line to the curve $y = 6x^3 + 5x - 3$ at the point $x = 0$.

A. $y = 5x - 3$
B. $y = \frac{1}{5}x - 3$
C. $y = -\frac{1}{5}x - 3$
D. $y = -5x - 3$

$x = 0,\ y = -3$
$y' = 18x^2 + 5$
$y'(0) = 5 \leftarrow$ slope tangent line
$y'(0) = -\frac{1}{5} \leftarrow$ slope normal line

$y + 3 = -\frac{1}{5}(x) \Rightarrow y = -1/5x - 3$

23. Let f and g be differentiable functions with the following properties:

(i) $g(x) > 0$ for all x
(ii) $f(0) = 1$

If $h(x) = f(x)g(x)$ and $h'(x) = f(x)g'(x)$ then $f(x)$=?

A. $f'(x)$ B. $g(x)$ C. 0   D. 1

$h'(x) = f(x)g'(x) + g(x) \cdot f'(x)$

must equal zero

24. A particle moves along the x-axis so that its position at time t is given by $x(t) = t^3 - 6t + 5$. For what value of t is the velocity of the particle zero?

A. 1 B. 2 C. 3 D. 4

25. If $f(x) = 3x^2 + \frac{1}{x}$ then what is $f'(x)$?

A. $f(x) = 6x + 1$ B. $f(x) = 6x - \frac{1}{x}$

C. $f(x) = 6x - \frac{1}{x^2}$ D. $f(x) = 6 - \frac{1}{x^2}$

$f(x) = 3x^2 + x^{-1}$

$f'(x) = 6x - x^{-2}$

$f'(x) = 6x - \frac{1}{x^2}$

26. If $f(x) = \frac{x}{2}(4x - 12)$ then what is the value of $f'(\frac{1}{2})$?

A. -4 B. -2 C. $\frac{5}{2}$ D. 4

$f(x) = \frac{4x^2}{2} - \frac{12x}{2}$

$f(x) = 2x^2 - 6x$

$f'(x) = 4x - 6$

$f'(\frac{1}{2}) = 2 - 6 = -4$

27. If $y = \frac{x}{x+1}$ then what is $\frac{dy}{dx}$?

A. 1 B. $\frac{2x+1}{(x+1)^2}$

C. $\frac{1}{(x+1)^2}$ D. $\frac{2x-1}{(x+1)^2}$

$\frac{dy}{dx} = \frac{(x+1)(1) - x(1)}{(x+1)^2} = \frac{x+1-x}{(x+1)^2} = \frac{1}{(x+1)^2}$

28. Given, $f(x) = \begin{cases} 4x^2 + x, & x \le -2 \\ |7 - 2x|, & -2 < x < 5 \\ \sqrt{x+4}, & x > 5 \end{cases}$ What is the $\lim_{x \to 5} f(x)$?

A. -3
B. 3
C. 105
D. does not exist

$\lim_{x \to 5^+} f(x) = \sqrt{5+4} = 3$

$\lim_{x \to 5^-} f(x) = |7 - 2(5)| = |-3| = 3$

29. Determine the x-coordinates of the point where the tangent line to the curve $y = \frac{4}{3}x^3 - 12x + 5$ are horizontal. slope zero

$y' = 4x^2 - 12$

$0 = 4x^2 - 12$

$\frac{0}{4} = \frac{4(x^2-3)}{4}$

$0 = x^2 - 3$

$x^2 = 3$

$x = \pm\sqrt{3}$

A. $x = 0, \pm\sqrt{3}$

B. $x = \pm\sqrt{3}$

C. $x = \pm 3$

D. $x = 0, \pm 3$

30. Given $f(x) = \frac{4}{(3x+1)^5}$, what is $f'(x)$?

$f(x) = 4(3x+1)^{-5}$

$f'(x) = -20(3x+1)^{-6}(3)$

$f'(x) = \frac{-60}{(3x+1)^6}$

A. $f'(x) = \frac{-60}{(3x+1)^6}$

B. $f'(x) = \frac{-20}{(3x+1)^6}$

C. $f'(x) = 20(3x+1)^4$

D. $f'(x) = 60(3x+1)^4$

31. What is the second derivative of $f(x) = (2x+1)^{\frac{5}{2}}$

$f'(x) = \frac{5}{2}(2x+1)^{3/2}(2)$

$f'(x) = 5(2x+1)^{3/2}$

$f''(x) = \frac{15}{2}(2x+1)^{1/2}(2)$

$f''(x) = 15(2x+1)^{1/2}$

A. $f''(x) = (2x+1)^{\frac{1}{2}}$

B. $f''(x) = \frac{15}{4}(2x+1)^{\frac{1}{2}}$

C. $f''(x) = \frac{5}{4}(2x+1)^{\frac{1}{2}}$

D. $f''(x) = 15(2x+1)^{\frac{1}{2}}$

32. Determine each of the following limits.

a) $\lim_{x\to 1}\left[\frac{2}{1-x^2} - \frac{1}{1-x}\right]$

b) $\lim_{x\to 5}\frac{x^2-25}{x^3-125}$

c) $\lim_{x\to 3}\left[\frac{\frac{4}{x+1} - \frac{1}{x-2}}{x-3}\right]$

d) $\lim_{x\to 0}\frac{\sqrt{x+1}-1}{2-\sqrt{4-x}}$

e) $\lim_{x\to\infty}\frac{5x-3}{\sqrt{x^2+2}}$

f) $\lim_{x\to\infty}\frac{2x^3+3x-1}{6x^2+4x-2}$

32 a) $\lim_{x \to 1} \left[\frac{2}{1-x^2} - \frac{1}{1-x} \right]$

$= \lim_{x \to 1} \left[\frac{2}{(1-x)(1+x)} - \frac{1}{1-x} \right]$

$= \lim_{x \to 1} \left[\frac{2}{(1-x)(1+x)} - \frac{(1+x)}{(1-x)(1+x)} \right]$

$= \lim_{x \to 1} \left[\frac{2-1-x}{(1-x)(1+x)} \right]$

$= \lim_{x \to 1} \left[\frac{\cancel{1-x}}{\cancel{(1-x)}(1+x)} \right]$

$= \frac{1}{2}$

difference of cubes
$x^3 - 125$
$x^3 - (5)^3$
$(x-5)(x^2+5x+25)$

b) $\lim_{x \to 5} \frac{x^2-25}{x^3-125}$

$= \lim_{x \to 5} \frac{\cancel{(x-5)}(x+5)}{\cancel{(x-5)}(x^2+5x+25)}$

$= \frac{10}{75}$

c) $\lim_{x \to 3} \left[\frac{\frac{4}{x+1} - \frac{1}{x-2}}{x-3} \right] = \lim_{x \to 3} \left(\frac{4(x-2)}{(x+1)(x-2)} - \frac{1(x+1)}{(x-2)(x+1)} \right) \cdot \frac{1}{x-3}$

$\lim_{x \to 3} \left(\frac{4x-8-x-1}{(x+1)(x-2)} \right) \cdot \left(\frac{1}{x-3} \right)$

$\lim_{x \to 3} \left(\frac{3x-9}{(x+1)(x-2)} \right) \cdot \frac{1}{x-3} = \lim_{x \to 3} \frac{3\cancel{(x-3)}}{(x+1)(x-2)} \cdot \left(\frac{1}{\cancel{x-3}} \right)$

$= \frac{3}{(4)(1)} = \frac{3}{4}$

32 d)

$$\lim_{x\to 0}\left[\frac{\sqrt{x+1}\;-1}{2-\sqrt{4-x}}\right]\cdot\left(\frac{\sqrt{x+1}\;+1}{\sqrt{x+1}\;+1}\right)\left(\frac{2+\sqrt{4-x}}{2+\sqrt{4-x}}\right)$$

$$\lim_{x\to 0}\frac{((x+1)-1)(2+\sqrt{4-x})}{[4-(4-x)]\;(\sqrt{x+1}\;+1)}$$

$$\lim_{x\to 0}\frac{(\cancel{x})(2+\sqrt{4-x})}{(\cancel{x})(\sqrt{x+1}\;+1)} = \frac{2+\sqrt{4}}{\sqrt{1}+1} = \frac{4}{2} = 2$$

e) $$\lim_{x\to\infty}\frac{5x-3}{\sqrt{x^2+2}} = \lim_{x\to\infty}\frac{5x-3}{\sqrt{x^2(1+2/x^2)}} = \lim_{x\to\infty}\frac{5x-3}{\sqrt{x^2}\sqrt{1+2/x^2}}$$

$$\lim_{x\to\infty}\frac{5x-3}{|x|\sqrt{1+2/x^2}} = \lim_{x\to\infty}\frac{5x-3}{x\sqrt{1+2/x^2}}$$

$$\lim_{x\to\infty}\frac{\frac{5x}{x}-3/x}{\frac{x}{x}\sqrt{1+2/x^2}} = \frac{5-0}{1\sqrt{1+0}} = \frac{5}{1\sqrt{1}} = 5$$

f). $$\lim_{x\to\infty}\frac{2x^3+3x-1}{6x^2+4x-2} = \lim_{x\to\infty}\frac{\frac{2x^3}{x^2}+\frac{3x}{x^2}-\frac{1}{x^2}}{\frac{6x^2}{x^2}+\frac{4x}{x^2}-\frac{2}{x^2}}$$

$$\lim_{x\to\infty}\frac{2x+\overset{\to 0}{3/x}-\overset{\to 0}{1/x^2}}{6+\underset{\to 0}{4/x}-2/x^2 \to 0} = \infty$$

32g) $\lim_{x\to\infty}\left[\dfrac{\frac{x}{x+3}+\frac{1}{2}}{x+1}\right] = \lim_{x\to 0}\left(\dfrac{\frac{2x}{2(x+3)}+\frac{x+3}{2(x+3)}}{x+1}\right)$

$\lim_{x\to\infty}\left[\dfrac{3x+3}{2(x+3)}\cdot\dfrac{1}{x+1}\right] = \lim_{x\to\infty}\dfrac{3\cancel{(x+1)}}{2(x+3)}\cdot\dfrac{1}{\cancel{(x+1)}}$

$\lim_{x\to\infty}\dfrac{3}{2x+6} = 0$

32h) $\lim_{x\to 3^+}\dfrac{|3-x|}{2x^2-5x-3} = \lim_{x\to 3^+}\dfrac{|-1(x-3)|}{2x^2-5x-3}$

$\lim_{x\to 3^+}\dfrac{|\cancel{x}-3|}{(2x+1)(x-3)} = \lim_{x\to 3^+}\dfrac{\cancel{(x-3)}}{(2x+1)\cancel{(x-3)}} = \dfrac{1}{7}$

33. $f(x) = \dfrac{(2x-1)^2}{4x^2-1} = \dfrac{(2x-1)(2x-1)}{4x^2-1} = \dfrac{(2x-1)(2x-1)}{(2x-1)(2x+1)} = \dfrac{2x-1}{2x+1}$

VA: $2x+1=0$

$\boxed{x = -\frac{1}{2}}$

$\left.\begin{aligned} \lim_{x\to -\frac{1}{2}^+}\left(\frac{2x-1}{2x+1}\right) &= -\infty \\ \lim_{x\to -\frac{1}{2}^-}\left(\frac{2x-1}{2x+1}\right) &= \infty \end{aligned}\right\}$

HA:

$\boxed{y=1}$

$\lim_{x\to\pm\infty}\dfrac{4x^2-4x+1}{4x^2-1} = \lim_{x\to\pm\infty}\dfrac{\frac{4x^2}{x^2}-\frac{4x}{x^2}+\frac{1}{x^2}}{\frac{4x^2}{x^2}-\frac{1}{x^2}} = 1$

34.

$$f(x) = \begin{cases} \dfrac{1-\sqrt{x-1}}{x-2} & x \neq 2 \\ K & x = 2 \end{cases}$$

Continuity

1) $f(2) = K$

2) $$\lim_{x \to 2} f(x) = \lim_{x \to 2} \left(\frac{1-\sqrt{x-1}}{x-2} \cdot \frac{1+\sqrt{x-1}}{1+\sqrt{x-1}} \right)$$

$$= \lim_{x \to 2} \left(\frac{1-(x-1)}{(x-2)(1+\sqrt{x-1})} \right)$$

$$= \lim_{x \to 2} \frac{(2-x)}{(x-2)(1+\sqrt{x-1})}$$

$$= \lim_{x \to 2} \frac{-\cancel{(x-2)}}{\cancel{(x-2)}(1+\sqrt{x-1})}$$

$$= -\frac{1}{2}$$

3) $$\lim_{x \to 2} f(x) = f(2)$$

$$\boxed{-\frac{1}{2} = K}$$

35. $f(x) = \begin{cases} x^3 - x + 1, & x < 1 \\ 0 & x = 1 \\ \dfrac{\sqrt{x+3}}{2} & x > 1 \end{cases}$

a) Continuous at $x=1$

1) $f(1) = 0$

2) $\lim\limits_{x\to 1} f(x) = \left.\begin{array}{l} \lim\limits_{x\to 1^+} \dfrac{\sqrt{x+3}}{2} = 1 \\ \lim\limits_{x\to 1^-} (x^3 - x + 1) = 1 \end{array}\right\} \lim\limits_{x\to 1} f(x) = 1$

3) $\lim\limits_{x\to 1} f(x) \neq f(x)$ not continuous at $x=1$

b) Differentiable at $x=1$

1) $f(1) = 0$

2) $f'(1) = \lim\limits_{x\to 1} \dfrac{f(x) - f(1)}{x-1} = \lim\limits_{x\to 1} \dfrac{f(x) - 0}{x-1} = \lim\limits_{x\to 1} \dfrac{f(x)}{x-1}$

$$\left.\begin{array}{l} \lim\limits_{x\to 1^+} \left(\dfrac{\sqrt{x+3}}{2} \cdot \dfrac{1}{x-1}\right) = \lim\limits_{x\to 1^+} \dfrac{\sqrt{x+3}}{2(x-1)} = \dfrac{2}{0} = +\infty \\ \lim\limits_{x\to 1^-} \dfrac{x^3 - x + 1}{x-1} = \dfrac{1}{0} = -\infty \end{array}\right\} \text{DNE}$$

Not differentiable at $x=1$

36. $f(x) = \begin{cases} \dfrac{3x+6}{x^2-x-6} & x \le 0 \\ \dfrac{x+5}{x-5} & x > 0 \end{cases}$

$\dfrac{3(x+2)}{(x-3)(x+2)}$ $x=-2$, $x=3$

$\dfrac{x+5}{x-5}$ $x=5$

Continuity

check $x=0$, $x=-2$, $x=5$

<u>$x=0$</u>

$f(0) = \dfrac{6}{-6} = -1$

$\left.\begin{aligned} &\lim_{x\to 0^+} \frac{x+5}{x-5} = -1 \\ &\lim_{x\to 0^-} \frac{3(x+2)}{(x-3)(x+2)} = -1 \end{aligned}\right\} \lim_{x\to 0} f(x) = -1$

$\lim_{x\to 0} f(x) = f(0)$

<u>Yes Continuous</u> at $x=0$

<u>$x=-2$</u>

$f(-2) = \dfrac{0}{0}$ undefined

$\lim_{x\to -2} \dfrac{3\cancel{(x+2)}}{(x-3)\cancel{(x+2)}} = -3/5$

not continuous at $x=-2$

<u>removable</u> discontinuity

<u>$x=5$</u>

$f(5) = \dfrac{10}{0}$ undefined

$\lim_{x\to 5} \dfrac{x+5}{x-5} = \text{DNE}$

not continuous at $x=5$

<u>non removable</u> discontinuity

g) $\lim_{x\to\infty} \dfrac{\dfrac{x}{x+3}+\dfrac{1}{2}}{x+1}$

h) $\lim_{x\to 3^+} \dfrac{|3-x|}{2x^2-5x-3}$

33. Given $f(x)=\dfrac{(2x-1)^2}{4x^2-1}$

State the equations of all the vertical and horizontal asymptotes of the function

34. Given $f(x)=\begin{cases}\dfrac{1-\sqrt{x-1}}{x-2}, & x\neq 2\\ k, & x=2\end{cases}$

Use the definition of continuity to determine the value of k such that the following function is continuous at $x = 2$.

35. Given $f(x)=\begin{cases}x^3-x+1, & \text{if } x<1\\ 0, & \text{if } x=1\\ \dfrac{\sqrt{x+3}}{2}, & \text{if } x>1\end{cases}$

(a) Determine whether $f(x)$ is continuous at $x = 1$.
(b) Is f differentiable at $x = 1$? Explain why or why not.

36. Use the definition of continuity to determine all points at which f is not continuous. Classify any discontinuities as removable or non-removable.

$$f(x)=\begin{cases}\dfrac{3x+6}{x^2-x-6}, & x\leq 0\\ \dfrac{x+5}{x-5}, & x>0\end{cases}$$

#37. $y = x^3 - 3x$

$y' = 3x^2 - 3$

parallel to x-axis $\Rightarrow m = 0$

$3x^2 - 3 = 0$

$3(x^2 - 1) = 0$

$x^2 - 1 = 0$

$x = \pm 1$

$(1, -2)$ and $(-1, 2)$

#38 a) $y = x^2 + (x^2-1)^5$

$y' = 2x + 5(x^2-1)^4(2x)$

$y' = 2x + 10x(x^2-1)^4$

b) $y = (x^4 - x + 1)^2 (x^2-2)^3$

$y' = (x^4 - x + 1)^2 \, (3(x^2-2)^2 \cdot (2x)) + (x^2-2)^3 \, (2(x^4-x+1)(4x^3-1$

$y' = 6x(x^4-x+1)^2(x^2-2)^2 + (8x^3-2)(x^4-x+1)(x^2-2)^3$

$y' = 6x(x^4-x+1)^2(x^2-2)^2 + 2(4x^3-1)(x^4-x+1)(x^2-2)^3$

$y' = 2(x^4-x+1)(x^2-2)^2 [3x(x^4-x+1) + (4x^3-1)(x^2-2)]$

$y' = 2(x^4-x+1)(x^2-2)^2 [3x^5 - 3x^2 + 3x + 4x^5 - 8x^3 - x^2 + 2]$

$y' = 2(x^4-x+1)(x^2-2)^2 [7x^5 - 8x^3 - 4x^2 + 3x + 2]$

38c) $f(x) = \frac{3}{4x^2} - \frac{4}{(3x-1)^2} + \frac{1}{(5x)^{-1}}$

$f(x) = \frac{3}{4}x^{-2} - 4(3x-1)^{-2} + 5x$

$f'(x) = -\frac{6}{4}x^{-3} + 8(3x-1)^{-3}(3) + 5$

$f'(x) = -\frac{6}{4}x^{-3} + 24(3x-1)^{-3} + 5$

$f'(x) = \frac{-6}{4x^3} + \frac{24}{(3x-1)^3} + 5$

38d) $f(x) = \sqrt{x + \sqrt{x^2+1}}$

$f(x) = (x + (x^2+1)^{\frac{1}{2}})^{\frac{1}{2}}$

$f'(x) = \frac{1}{2}(x + (x^2+1)^{\frac{1}{2}})^{-1/2} \cdot [1 + \frac{1}{2}(x^2+1)^{-1/2} \cdot 2x)]$

$f'(x) = \frac{1}{2\sqrt{x+\sqrt{x^2+1}}} \cdot \left[1 + \frac{x}{\sqrt{x^2+1}}\right]$

38e) $y = \frac{1}{\sqrt[3]{1-x^4}} = \frac{1}{(1-x^4)^{1/3}} = (1-x^4)^{-1/3}$

$y' = -1/3(1-x^4)^{-4/3}(-4x^3)$

$y' = \frac{4x^3}{3\sqrt[3]{1-x^4}}$

38 f). $y = \left(\frac{2x-1}{x+2}\right)^6$

$$y' = 6\left(\frac{2x-1}{x+2}\right)^5 \left[\frac{(x+2)(2) - (2x-1)(1)}{(x+2)^2}\right]$$

$$y' = 6\left(\frac{2x-1}{x+2}\right)^5 \left[\frac{2x+4-2x+1}{(x+2)^2}\right]$$

$$y' = 6\left(\frac{2x-1}{x+2}\right)^5 \left[\frac{5}{(x+2)^2}\right]$$

$$y' = \frac{30(2x-1)^5}{(x+2)^3}$$

39. $f(x) = \sqrt{1+x^3} = (1+x^3)^{\frac{1}{2}}$

$$f'(x) = \frac{1}{2}(1+x^3)^{-1/2}(3x^2)$$

$$f'(x) = \frac{3}{2}x^2(1+x^3)^{-1/2}$$

$$f''(x) = \frac{3}{2}\left[(x^2) -\frac{1}{2}(1+x^3)^{-3/2}(3x^2) + (1+x^3)^{-1/2}(2x)\right]$$

$$f''(x) = \frac{3}{2}\left[\frac{-3}{2}x^4(1+x^3)^{-3/2} + 2x(1+x^3)^{-1/2}\right]$$

$$f''(x) = \frac{3}{2}x(1+x^3)^{-3/2}\left[\frac{-3}{2}x^3 + 2(1+x^3)\right]$$

40. $y = \frac{1}{\sqrt{20-x^4}}$ at $x=2$

$x=2$
$y=\frac{1}{2}$

$y = (20-x^4)^{-\frac{1}{2}}$

$y' = -\frac{1}{2}(20-x^4)^{-3/2}(-4x^3)$

$y' = \frac{2x^3}{(20-x^4)^{3/2}}$

$y'(2) = \frac{16}{8} = 2$

$y - \frac{1}{2} = 2(x-2)$

41. $xy^3 + xy = 6 \quad (3,1)$

$x(3y^2)\frac{dy}{dx} + y^3 + x\frac{dy}{dx} + y(1) = 0$

$\frac{dy}{dx}(3xy^2 + x) = -y^3 - y$

$\frac{dy}{dx} = \frac{-y^3-y}{3xy^2+x} = \frac{-(1)^3-(1)}{3(3)(1)+3} = \frac{-2}{9+3} = \frac{-1}{6}$

$y - 1 = \frac{-1}{6}(x-3)$

42. $x^3 + y^3 = 9$

$$3x^2 + 3y^2 \cdot \frac{dy}{dx} = 0$$

$$3y^2 \frac{dy}{dx} = -3x^2$$

$$\frac{dy}{dx} = -\frac{x^2}{y^2}$$

$$\frac{d^2y}{dx^2} = \frac{y^2(-2x) - (-x^2)(2y \cdot dy/dx)}{y^4}$$

$$\frac{d^2y}{dx^2} = \frac{-2xy^2 + 2x^2y\, dy/dx}{y^4}$$

$dy/dx = -\frac{x^2}{y^2}$

$$\frac{d^2y}{dx^2} = \frac{-2xy^2 + 2x^2y\left(-\frac{x^2}{y^2}\right)}{y^4}$$

$$\frac{d^2y}{dx^2} = \frac{-2xy^2 - \frac{2x^4}{y}}{y^4}$$

37. Find all points on the graph of $y = x^3 - 3x$ where the tangent line is parallel to the x axis.

38. Use appropriate rules to determine $f'(x)$ for each of the following.

(a) $y = x^2 + (x^2 - 1)^5$

(b) $y = (x^4 - x + 1)^2 (x^2 - 2)^3$

(c) $f(x) = \dfrac{3}{4x^2} - \dfrac{4}{(3x-1)^2} + \dfrac{1}{(5x)^{-1}}$

d) $f(x) = \sqrt{x + \sqrt{x^2 + 1}}$

e) $y = \dfrac{1}{\sqrt[3]{1 - x^4}}$

f) $y = \left(\dfrac{2x-1}{x+2}\right)^6$

39. If $f(x) = \sqrt{1 + x^3}$, determine $f''(2)$.

40. Determine the equation of the tangent line to the curve $y = \dfrac{1}{\sqrt{20 - x^4}}$ at $x = 2$.

41. Determine the equation of the tangent line to the curve $xy^3 + xy = 6$ at the point (3,1).

42. If $x^3 + y^3 = 9$, use implicit differentiation to find y''.