Section 6.2 Derivatives of Trigonometric Functions

↳ $f'(x)$ is the slope of the tangent line at $(x, f(x))$

Remember:

$\sin(A+B) = \sin A\cos B + \cos A\sin B$

$\cos(A+B) = \cos A\cos B - \sin A\sin B$

Example 1

Calculate $f'(x)$ where $f(x) = \sin x$

x is in radians

Algebra

Graph

(section 2.3 animation)

$$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

→

Example 2

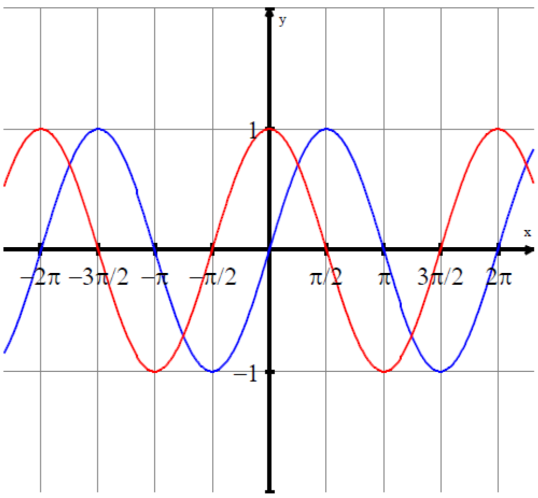
Calculate $f'(x)$ where $f(x) = \cos x$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

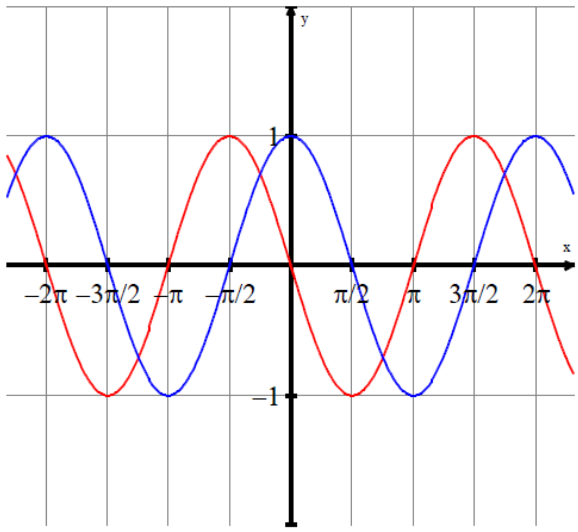
→

Graphical Comparisons

$y = \sin x$ and $\frac{dy}{dx} = \cos x$



$y = \cos x$ and $\frac{dy}{dx} = -\sin x$



Example 3

Calculate $f'(x)$ where $f(x) = \tan x$

Example 4

Calculate $f'(x)$ where $f(x) = \csc x$

Summary

Derivatives of Trigonometric Functions

$$\frac{d}{dx}(\sin x) = \cos x \qquad \frac{d}{dx}(\csc x) = -\csc x \cot x$$

$$\frac{d}{dx}(\cos x) = -\sin x \qquad \frac{d}{dx}(\sec x) = \sec x \tan x$$

$$\frac{d}{dx}(\tan x) = \sec^2 x \qquad \frac{d}{dx}(\cot x) = -\csc^2 x$$

Example 5

Determine the derivatives of the following:

(a) $y = 3\sin x$

(b) $y = \dfrac{\sin x}{x^2}$

→

(c) $y = 3\sin x + \cos x - 1$

(d) $y = (\sec x)(\csc x)$

(e) $y = \frac{1}{x} - 2\pi x - \frac{\sin x}{2}$

(f) $y = \frac{1 - \sec x}{\tan x}$

→

Derivatives of Trigonometric Functions

- power rule/product rule/quotient rule combined with chain rule
- implicit differentiation

Chain Rule

$$\frac{d}{dx}(\sin u) = \cos u \frac{du}{dx} \qquad \frac{d}{dx}(\csc u) = -\csc u \cot u \frac{du}{dx}$$

$$\frac{d}{dx}(\cos u) = -\sin u \frac{du}{dx} \qquad \frac{d}{dx}(\sec u) = \sec u \tan u \frac{du}{dx}$$

$$\frac{d}{dx}(\tan u) = \sec^2 u \frac{du}{dx} \qquad \frac{d}{dx}(\cot u) = -\csc^2 u \frac{du}{dx}$$

Notational Issues to Watch-out for:

$(\sin x)^2 = \sin^2 x$

$\sin x^2 = \sin(x)^2$

$\sin(\cos x) \neq \sin x \cos x$

Example 6

Determine the derivatives of the following:

(a) $y = 5\cos\left(\frac{x}{2}\right)$

(b) $y = \sin(x^3)$

$$\frac{d}{dx}(\sin u) = \cos u \frac{du}{dx}$$

(c) $y = \sin^3 x$

(d) $y = \tan\sqrt{1-x}$

→

(e) $y = 2\cos^3\sqrt{1-x}$

(f) $y = \sin^3\left(\dfrac{5}{2x-1}\right)$

(g) $y = \dfrac{\sin 2x}{1-\cos x}$

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(h) $y = \sec x - \tan^2 x$

(i) $y = 5\cos^3(1 + \tan x)$

(j) $y = \cot(3x^2 + 5)$

→

(k) $y = \sqrt{1 - 2\sin 3x}$

P. 112-113 #3-4, 7-10, 19-23, 29 (Product, Quotient)

P. 120-121 #10, 12, 23, 29-33 (Chain Rule)

Example 7

Determine $\frac{dy}{dx}$ for each of the following relations

(a) $x^2 \cos y = y^2 \sin x$

(b) $x\cos(xy) = y\sin(3x)$

p.127 #7-10, 13 (Implicit)

Example 8

Determine the equation of the tangent line of the relation

$\sin(xy - y^2) = x^2 - 1$ at the point (1,1)

→