Section 6.4 Inverse Trigonometric Functions

1. Graph the Inverse Trigonometric Functions
2. Evaluate Expressions involving an Inverse Trigonometric Function
3. Define the Derivatives of the Inverse Trigonometric Functions

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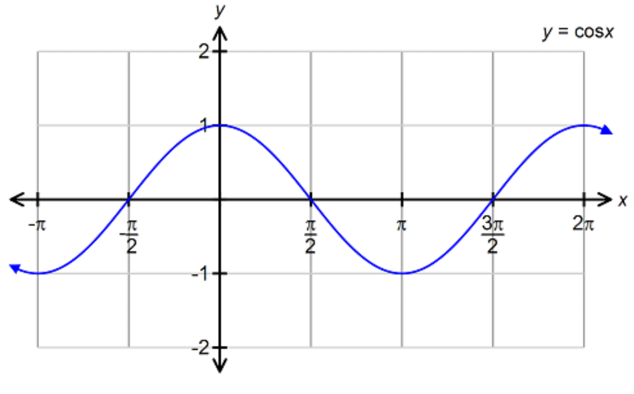
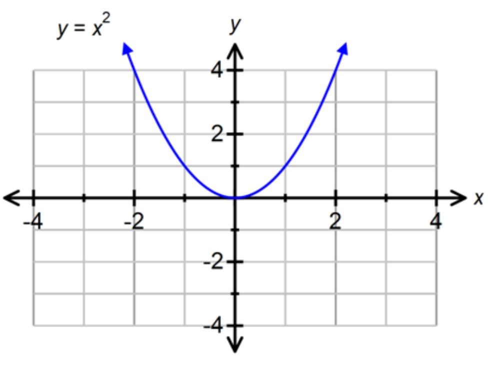
Review

The inverse of a relation ⟶ can be found by interchanging the elements of the ordered pairs of the relation

The inverse of a relation is a function ⟶ the original relation passed the horizontal line test





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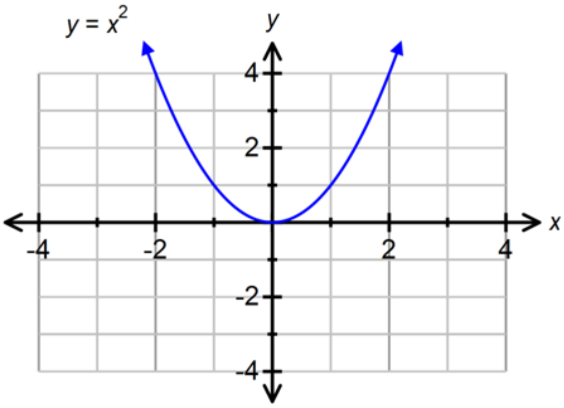
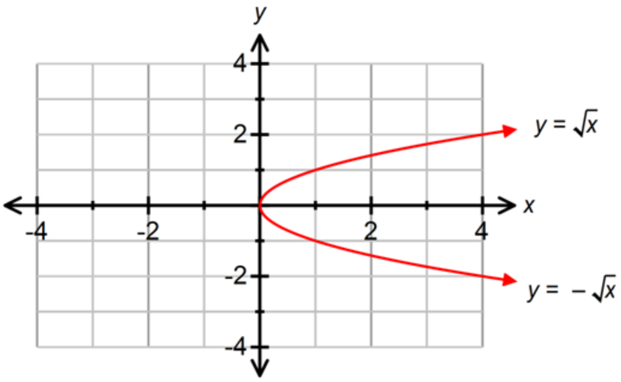
Review





Domain:

Range:

Equation: $y = x^2$

Domain:

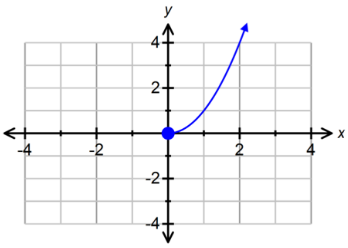
Range:

Equation: $y = \pm\sqrt{x}$

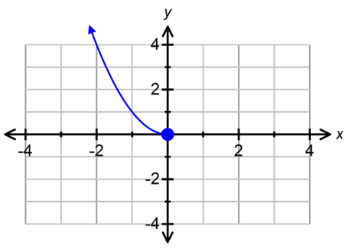
Is the graph of the inverse a function?

Restrict domain of original function → in order for the inverse to be a function

(i)



(ii)



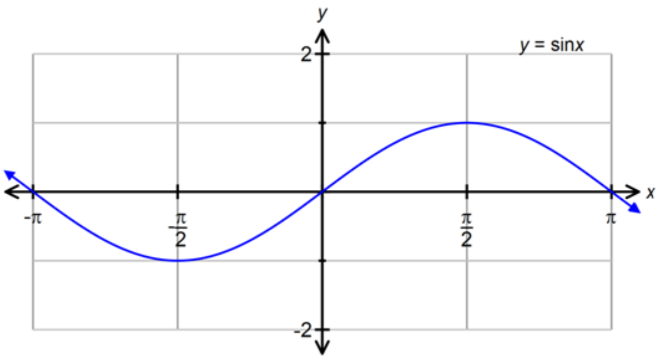
Notation

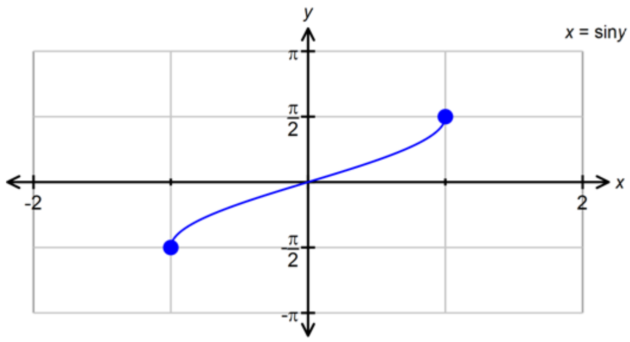
Do not confuse reciprocal trigonometric functions with inverse trigonometric functions

$\frac{1}{\sin x} \neq \sin^{-1} x$ $\qquad (\sin(x))^{-1} = \frac{1}{\sin x}$ $\qquad \sin^{-1} x = \arcsin(x)$

Inverse of the Sine Function: $y = \sin^{-1} x$ or $y = \arcsin(x)$

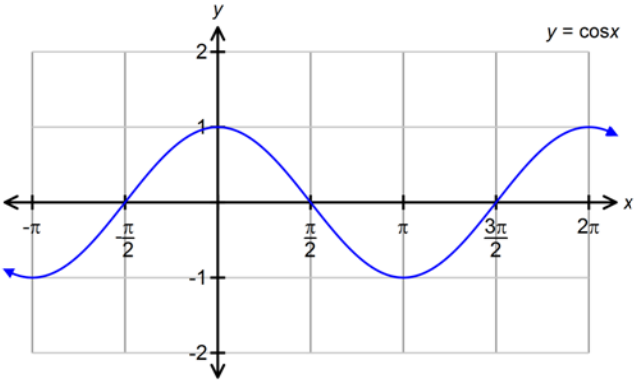
$$y = \sin^{-1}(x) \Leftrightarrow x = \sin y$$

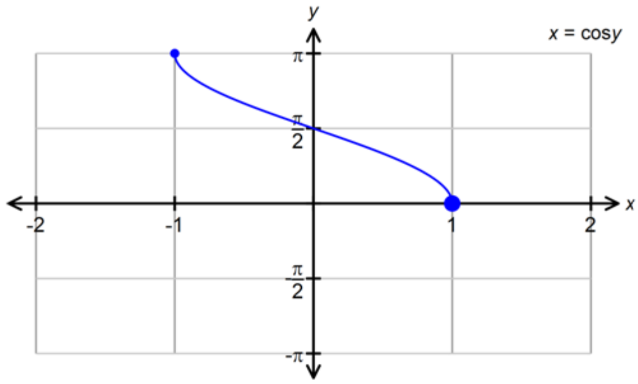




Restrict Domain:

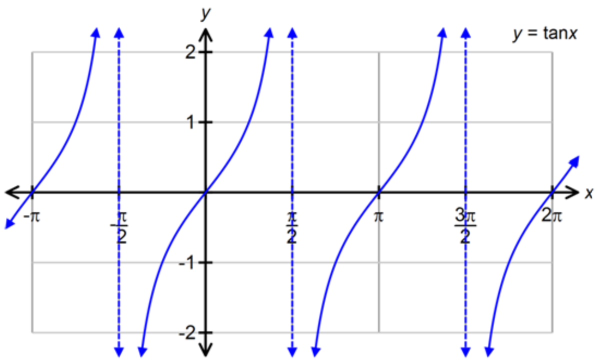
Inverse of the Cosine Function: $y = \cos^{-1} x$ or $y = \arccos(x)$

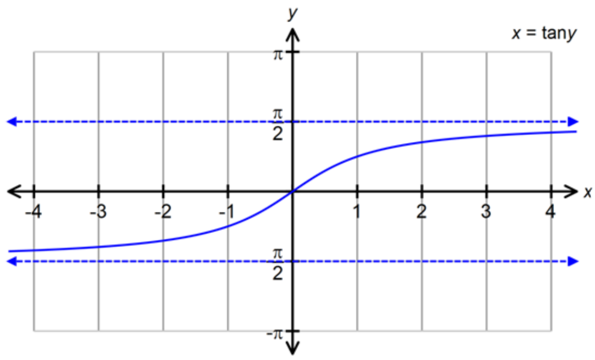




Restrict Domain:

Inverse of the Tangent Function: $y = \tan^{-1} x$ or $y = \arctan(x)$

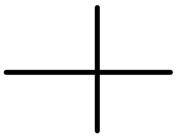




Restrict Domain:

Exact value of an expression involving an inverse trigonometric function

Review:

Solve $\sin\theta = \frac{1}{2}$	Evaluate $\sin^{-1}\left(\frac{1}{2}\right)$
continuous periodic graph	the graph has restrictions
$\sin^{-1}\left(\frac{1}{2}\right) = \theta$ no restrictions	$\sin\theta = \frac{1}{2}$ where $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
	
$\theta =$	$\theta =$

Examples

a) Determine the exact value of $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$.

→ $\sin\theta = \frac{\sqrt{3}}{2}$ where $[-\frac{\pi}{2}, \frac{\pi}{2}]$

b) Determine the exact value of $\arcsin\left(-\frac{\sqrt{3}}{2}\right)$.

c) Determine the exact value of $\tan\left(\cos^{-1}\frac{\sqrt{2}}{2}\right)$.

d) Determine the exact value of $\sin^{-1}\left(\sin\frac{\pi}{4}\right)$.

→

e) Determine the exact value of $\sin^{-1}\left(\sin \frac{7\pi}{6}\right)$.

f) Determine the exact value of $\arctan\left(\tan \frac{2\pi}{3}\right)$.

Practice Questions: P.297 #1-6

Inverse Trigonometric Derivatives

$\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$	$\frac{d}{dx}(\cos^{-1} x) = -\frac{1}{\sqrt{1-x^2}}$
$\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$	$\frac{d}{dx}(\cot^{-1} x) = -\frac{1}{1+x^2}$
$\frac{d}{dx}(\sec^{-1} x) = \frac{1}{\lvert x\rvert\sqrt{x^2-1}}$	$\frac{d}{dx}(\csc^{-1} x) = -\frac{1}{\lvert x\rvert\sqrt{x^2-1}}$

Reminder:

$y = \sin^{-1} x$ was created by restricting the domain of $y = \sin x$ to $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$

$y = \cos^{-1} x$ was created by restricting the domain of $y = \cos x$ to $0 \le x \le \pi$

$y = \tan^{-1} x$ was created by restricting the domain of $y = \tan x$ to $-\frac{\pi}{2} < x < \frac{\pi}{2}$

→

Proof

Derive the derivative of the inverse sine function: $\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$

Derive the derivative of the inverse tangent function: $\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$

→

Example 1

Determine $f'(x)$ for each of the following:

(a) $y = \tan^{-1}(3x)$

(b) $y = \cos^{-1}(2x+1)$

(c) $f(x) = \arcsin(x) + \arccos x$

→

(d) $f(x) = \cos^{-1}\sqrt{2x-1}$

(e) $f(x) = (\arctan x)^{-1}$

(f) $f(x) = x^2 \cos^{-1}\left(\frac{2}{x}\right)$

→

Example 2

Write the equation of the tangent line to $f(x) = x\sin^{-1}\left(\frac{x}{4}\right) + \sqrt{16 - x^2}$ at $x = 2$

Example 3

Write the equation of the tangent line to $y = \arccos\left(\frac{x}{2}\right)$ at $x = 1$

Practice Questions P.297 #16-18, 24, 25, 27