Unit: Anti-differentiation and Integration

- (i) Indefinite Integral of Polynomial and Radical Functions
- (ii) Definite Integral of a Polynomial Function

What is an Antiderivative?

Differentiation

$$x^4 \longrightarrow 4x^3 \longrightarrow 12x^2 \longrightarrow 24x$$

Anti-differentiation

$$24x \longrightarrow 12x^2 \longrightarrow 4x^3 \longrightarrow x^4$$

Anti-differentiation —— undoing the derivative

Definition: F is an antiderivative of f if F'(x) = f(x) for all x in the interval.

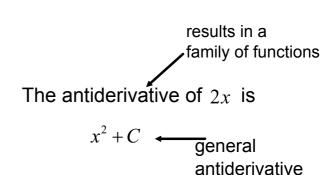
Example: What is the antiderivative of f(x) = 2x?

$$F(x) = x^2$$

$$F(x) = x^2 + 1$$

$$F(x) = x^2 - 4$$

possible antiderivatives



Rules for Antiderivatives

Consider:			Function	Antiderivative
$f(x) = x^2$	$F(x) = \frac{x^3}{3} + C$	-	$x^n, n \neq -1$	$\frac{x^{n+1}}{n+1} + C$
$f(x) = x^3$	$F(x) = \frac{x^4}{4} + C$		$f(x) \pm g(x)$	$F(x) \pm G(x) + C$
$f(x) = x^4$	$F(x) = \frac{x^5}{5} + C$		af(x)	aF(x) + C

Example 1: What are the general anti-derivatives for each of the following:

(i)
$$f(x) = 5$$

(ii)
$$f(x) = 4x$$

(iii)
$$f(x) = 3x^2$$

(iv)
$$f(x) = 7x^2 - 2x + 1$$

(v)
$$f(x) = -2x^5 - 4x^2$$

(vi)
$$f(x) = \frac{1}{x^2}$$

(Vii)
$$f(x) = \frac{1}{x}$$

section 8.1 Indefinite Integral

Integral Notation

Antidifferentiation

The antiderivative of f(x) = 10x is $F(x) = 5x^2 + C$

Indefinite Integration

$$\int 10x dx = 5x^2 + C$$
 The indefinite integral of $10x$ with respect to x is $5x^2 + C$

Table of Integration Rules

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C, n \neq -1$$

$$\int f(x) \pm g(x) \, dx = \int f(x) \, dx \pm \int g(x) \, dx$$

$$\int af(x) \ dx = a \int f(x) \ dx$$

$$\int k \, dx = kx + C$$

section 8.1 Indefinite Integral

Example 2: Integrate the following

(i)
$$\int 3\sqrt{x}dx$$

$$(ii) \int \frac{4}{\sqrt[3]{x^5}} dx$$

(iii)
$$\int (2x-1)^2 dx$$

(iv)
$$\int (6x^5 + 3x^2 - 2x + 12) dx$$

$$(v) \int \left(\frac{x^2-9}{x+3}\right) dx$$

$$(vi)\left(\frac{x+1}{x^3}\right)dx$$

Indefinite Integral given Extra Conditions

Example 3: Find f(x) if f'(x) = 2x+1 where f(0) = 3

Example 4: Find f(x) if $f''(x) = 12x^2 - 8$ and the slope of the tangent line at the point (2, 3) is -8.

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Example 5:The velocity of a ball being thrown in the air from an initial height of 1m is given by v(t) = -9.8t + 12 where t is the time in seconds. Determine the quadratic function that models the height of the ball after t seconds. Use this function to calculate the height of the ball after 2.5 seconds?

Example 6: A particle moves in a straight line and has acceleration given by a(t) = 12t. Determine its position at t = 2 if v(t) = 22 and x(0) = 0.

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