

Unit: Anti-differentiation and Integration

(i) Indefinite Integral of Polynomial and Radical Functions

(ii) Definite Integral of a Polynomial Function

What is an Antiderivative?

Differentiation

$$x^4 \longrightarrow 4x^3 \longrightarrow 12x^2 \longrightarrow 24x$$

Anti-differentiation

$$24x \longrightarrow 12x^2 \longrightarrow 4x^3 \longrightarrow x^4$$

Anti-differentiation \longrightarrow undoing the derivative

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Definition: F is an antiderivative of f if $F'(x) = f(x)$ for all x in the interval.

Example: What is the antiderivative of $f(x) = 2x$?

$$F(x) = x^2$$

$$F(x) = x^2 + 1$$

$$F(x) = x^2 - 4$$

possible antiderivatives

The antiderivative of $2x$ is $x^2 + C$ ← general antiderivative

↙ results in a family of functions

Rules for Antiderivatives

Consider:

$$f(x) = x^2 \quad F(x) = \frac{x^3}{3} + C$$

$$f(x) = x^3 \quad F(x) = \frac{x^4}{4} + C$$

$$f(x) = x^4 \quad F(x) = \frac{x^5}{5} + C$$

Function	Antiderivative
$x^n, n \neq -1$	$\frac{x^{n+1}}{n+1} + C$
$f(x) \pm g(x)$	$F(x) \pm G(x) + C$
$af(x)$	$aF(x) + C$

section 8.1 Indefinite Integral

Example 1: What are the general anti-derivatives for each of the following:

(i) $f(x) = 5$

(ii) $f(x) = 4x$

(iii) $f(x) = 3x^2$

(iv) $f(x) = 7x^2 - 2x + 1$

(v) $f(x) = -2x^5 - 4x^2$

(vi) $f(x) = \frac{1}{x^2}$

(vii) $f(x) = \frac{1}{x}$

Integral Notation

Antidifferentiation



The antiderivative of $f(x) = 10x$ is $F(x) = 5x^2 + C$

Indefinite Integration



$\int 10x dx = 5x^2 + C$ \longrightarrow The indefinite integral of $10x$ with respect to x is $5x^2 + C$

Table of Integration Rules

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C, n \neq -1$$

$$\int f(x) \pm g(x) dx = \int f(x) dx \pm \int g(x) dx$$

$$\int af(x) dx = a \int f(x) dx$$

$$\int k dx = kx + C$$



section 8.1 Indefinite Integral

Example 2: Integrate the following

$$(i) \int 3\sqrt{x} dx$$

$$(ii) \int \frac{4}{\sqrt[3]{x^5}} dx$$

$$(iii) \int (2x - 1)^2 dx$$

$$(iv) \int (6x^5 + 3x^2 - 2x + 12) dx$$

$$(v) \int \left(\frac{x^2 - 9}{x + 3} \right) dx$$

$$(vi) \int \left(\frac{x+1}{x^3} \right) dx$$

Indefinite Integral given Extra Conditions

Example 3: Find $f(x)$ if $f'(x) = 2x + 1$ where $f(0) = 3$

Example 4: Find $f(x)$ if $f''(x) = 12x^2 - 8$ and the slope of the tangent line at the point $(2, 3)$ is -8 .

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Example 5: The velocity of a ball being thrown in the air from an initial height of 1m is given by $v(t) = -9.8t + 12$ where t is the time in seconds. Determine the quadratic function that models the height of the ball after t seconds. Use this function to calculate the height of the ball after 2.5 seconds?

Example 6: A particle moves in a straight line and has acceleration given by $a(t) = 12t$. Determine its position at $t = 2$ if $v(t) = 22$ and $x(0) = 0$.

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