

Worksheet 5: Overview of Limits

1. Evaluate the following limits, if they exist. Assign ∞ or $-\infty$ where appropriate to any limit that does not exist.

$$(a) \lim_{x \rightarrow 3} \frac{x^2 - 9}{x + 1}$$

$$(b) \lim_{x \rightarrow 3} \frac{\sqrt{x+1} - 2}{3 - x}$$

$$(c) \lim_{x \rightarrow -4^+} \frac{4x - 6}{2x^2 + 5x - 12}$$

$$(d) \lim_{x \rightarrow 2} \frac{5(x+3)^{-1} - (x-1)^{-1}}{x-2}$$

$$(e) \lim_{x \rightarrow -5^+} \frac{x}{25 - x^2}$$

$$(f) \lim_{x \rightarrow 0} \frac{x^2 - 2|x|}{x}$$

$$(g) \lim_{x \rightarrow a} \frac{\sqrt{x} - \sqrt{a}}{x - a}$$

$$(h) \lim_{x \rightarrow 0} \left(\frac{3}{x} + \frac{12}{x^2 - 4x} \right)$$

$$(i) \lim_{x \rightarrow 2^+} \frac{|x^2 - 2x|}{2x - 4}$$

$$(j) \lim_{x \rightarrow 2} \frac{\sqrt{x} - \sqrt{2}}{\sqrt{x+2} - 2}$$

$$(k) \lim_{x \rightarrow \infty} \frac{5x^3 - 1}{x^3 + 3x^2 - 6}$$

$$(l) \lim_{x \rightarrow -\infty} \frac{4x^2 - x}{2x^3}$$

$$(m) \lim_{x \rightarrow 5} \frac{8x^2}{\sqrt{5-x}}$$

$$(n) \lim_{x \rightarrow 3^+} \frac{|3-x|}{x^2 - 6x + 9}$$

$$(o) \lim_{x \rightarrow 0} \left(\frac{1}{x} - \frac{1}{|x|} \right)$$

$$(p) \lim_{x \rightarrow 4} \frac{8-2x}{|x-4|}$$

2. Determine the vertical asymptote (using limits) of the function $f(x) = \frac{x^2 - 4}{x^2 - 3x + 2}$

3. Determine the horizontal asymptote (using limits) of the function $f(x) = \frac{x^2 + 3x - 2}{2x^2 + 4x - 5}$

4. Determine the $\lim_{x \rightarrow a} \frac{x^2 - a^2}{x^4 - a^4}$ where $a \neq 0$

5. Determine the vertical and horizontal asymptotes for $f(x) = \frac{\sqrt{x^2 + 9} - 5}{x^2 + 4x}$

6. If $f(x) = \frac{1}{x-2}$ and $\lim_{x \rightarrow (-k+1)} f(x)$ does not exist, determine the value of k .

7. For some unknown function $f(x)$, we are told that $\lim_{x \rightarrow 0} \frac{f(x)-3}{5x} = 7$.

Can $\lim_{x \rightarrow 0} f(x)$ be determined using only this information? Explain your answer.